Theorem 5.2 *Let $(\Omega, \mathcal{A}, P)$ be a probability space and let X be an integrably bounded fuzzy random variable. Then, there exists a sequence of integrably bounded fuzzy random variables $\{X_m\}_m$ with simple α-levels functions such that*

$$\lim_{m \to \infty} d_\infty(\tilde{E}_P(X_m), \tilde{E}_P(X)) = 0. \quad \square$$

6 Concluding remarks

As we have commented before, the metric spaces $(\mathcal{K}(\mathbb{R}^p), d)$ and $(\mathcal{F}(\mathbb{R}^p), d_\infty)$ are complete. Assume that we now add a convexity condition to subsets in $\mathcal{K}(\mathbb{R}^p)$ and $\mathcal{F}(\mathbb{R}^p)$, so that $\mathcal{K}_c(\mathbb{R}^p)$ denotes the collection of nonempty compact convex subsets of $\mathbb{R}^p$, and $\mathcal{F}_c(\mathbb{R}^p)$ is the class of fuzzy sets of $\mathbb{R}^p$ being upper semicontinuous, normal, convex and having compact closed convex hull of the support. Then, $(\mathcal{K}_c(\mathbb{R}^p), d)$ is a closed subspace of $(\mathcal{K}(\mathbb{R}^p), d)$, and $(\mathcal{F}_c(\mathbb{R}^p), d_\infty)$ is a closed subspace of $(\mathcal{F}(\mathbb{R}^p), d_\infty)$.

The results in Sections 3, 4 and 5 can be established for $\mathcal{F}_c(\mathbb{R}^p)$-valued fuzzy random variables. In this case, both variables, the limit one and those in the convergent sequence, will be $\mathcal{F}_c(\mathbb{R}^p)$-valued.

It should be emphasized that, due to the added convexity, the Aumann integral of a simple random set can be easily computed by means of the use of the scalar product and the Minkowski addition (see Debreu, 1966, Byrne, 1978). Consequently, the fuzzy expected value of a simple fuzzy random variable can be easily computed by means of the use of the fuzzy product by a scalar and the fuzzy addition based on Zadeh's extension principle (1975).

If we combine the last conclusions with Theorems 5.1 and 5.2, we can obtain an operational procedure to compute the fuzzy expected value of a general integrably bounded fuzzy random variable.

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Is it possible the agreement between Bayesian Theory and Fuzzy Sets ?

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Abstract— **Everybody in the *fuzzy* field knows about the controversy between Probability and Fuzziness. Some articles are still published from time to time, discussing conceptual differences and showing comparative analyses or collaborative approaches. In this paper we point out what we think is the main issue, sometimes made obscure because of accompanying complains: a pure Bayesian does not accept the idea of modeling other thing different than sets, which are always crisp.**

Key words: Fuzzy Sets Theory, Decision Making, Probability Theory.

I. Introduction.

Since Zadeh's foundational paper on Fuzzy Sets [21], doubts about the relevance of the *fuzzy* model as an alternative model for uncertainty have been made patent by some key probability researchers, mainly coming from the Bayesian field.

The two authors of this article devoted some long friendly conversations in order to get a better understanding about such a controversy. Being each one of them working on a different side, that was an advantage. This paper reflects part of their final conclusion: in no way the controversy is due to a number of small arguments, but to a deep different point of view. A pure Bayesian, even if he/she is willing to accept the existence of different kinds of uncertainties, will still claim that only crisp based models deserve to be developed [11]. Listing detailed descriptions of advantages or disadvantages of each approach is sometimes misleading the main issue.

II. A short view to Bayesian postulates.

The controversy between Fuzzy Sets and Probability Theory comes in fact from the very beginning of the history of Fuzzy Sets Theory. Dubois and Prade [5], for example, refer to a very short paper by Loginov [10], submitted for publication as early as December 24 (1965). Since then, Fuzzy Set Theory has been the focus of a sometimes strong controversy. Articles about the relationship between fuzziness and probability are still being published (see, e.g., [5] or the special issue of the IEEE Transactions on Fuzzy Systems -vol. 2(1), February 1994).

A pure Bayesian claims that Probability is all we need. Let us remind main Bayesian postulates following some key papers, all of them very well-known in the fuzzy literature: Cheesman [1], French [6], Laviolette-Seaman [6], Lindley [9], Natvig [12] and Stallings [20].

First of all, Bayesian Theory use to be presented as a *subjective* theory for Probability, opposed to the *frequentist* approach. In fact, some arguments initially coming from the fuzzy field did apply only to a frequentist approach to probability [12,20]).

The term *subjective* should be rightly understood, since Bayesian Theory pursues in fact objectivity. If we want to build up a model in order to deal with uncertainty. Bayesians say, we have to use only available information, not assumptions. The use of *prior* knowledge in the Bayesian model should not make us think that Bayesian Theory is *not objective*. This would be the case if those *priors* are defined without justification. Decision maker personal experience and opinions may not be directly known. But we do know about the decisions such a decision maker makes on the basis of them. We can look at his/her *acts*. Acts will allow Bayesian Theory to be *operational*. Bayesian Theory indeed deals with *personal* activities, but it tries in fact to be *objective* in every aspect. This key argument is made clear, e.g., by Lindley [9]. the Bayesian approach is the only one defining a measure of uncertainty being "operationally testable" (Laviolette-Seaman [8]). Hence, Bayesian Probability is the only way of operationally dealing with uncertainty. The controversy should not be understood as fuzziness versus probability. There is no particular controversy between Bayesian Theory versus fuzziness. The controversy is Bayesian approach versus any other model for uncertainty.

Related to Lindley's arguments are Cox's papers [3,4]. Cox's work shows how basic rules of probability follow from consistence with Boolean algebra, where a key role is played by the basic Aristotelian assumption which classifies every assertion in one and only one of two classes: true and false (see Zadeh [22]). Therefore, from this binary logic we reach probability, and from probability we'll fall into the Bayesian model. That's the underlying Bayesian argument in Cheeseman [1].

Can a pure Bayesian model be built up from a non-binary logic?

Bayesian Theory claims for *rationality* in decision making (see, e.g., French [6]), because personal decision making is considered as the only operational approach to uncertainty (see, e.g., Laviolette-Seaman [8]). Some basic Bayesian assumption can be criticized, but it is not possible to make rational decisions without making such assumptions, points out Cheeseman [1] (see also [2]). But in order to be *operational*, Bayesians need a *crisp* space of actions.

Pure Bayesians do not care about what the term *probability* means in the real world. They do not care about anything called *probabilistic uncertainty*. They do not care if concepts are *crisp* or *fuzzy*. They do not like arguments like "the world is fuzzy, therefore our mathematics should also be fuzzy" (see French [6]). We do not know about arguments, we just know about acts, they claim. Crispness assumption in the Bayesian model applies to *acts*, and the Bayesian model refers just to *acts*.

Acts are always crisp. Only occurrence of (crisp) acts can be checked. This is a key Bayesian argument.

Bayesian probability is a prescriptive theory, not a descriptive one. Bayesians do not pursue explaining how human beings think, but how to model consistent decision making. If you have been coherent in your behavior, your preferences can be modeled as if you had a Bayesian mind. From a Bayesian point of view, fuzziness should be understood just as uncertainty about meaning. Some Bayesians claim in fact that any kind of *uncertainty* can be perfectly represented as a probability distribution defined on the family of all possible intended meanings (Cheeseman [1]). But the question about how *perfect* is this representation is not the real issue here. A pure Bayesian will claim that this is the only choice, as a consequence of the *operationality* need. Occurrence of those intended meanings are made testable when all those possible intended meanings are imposed to be *crisp*. Crisp acts are the only possible foundation for a *rational* decision making theory.

Bayesian Theory indeed models *personal* but *consistent* behavior, which refers to decision making about crisp acts. It is understood as an operational theory, since behavior can be measured by means of acts and bets. Bayesian Theory conclude that our behavior -if consistent- can be solely modeled as if we had in our mind a probabilistic distribution on the set of acts, as if we had a *probabilistic opinion*. How come an individual reaches to a decision is not observable: final acts are the only observable information.

Bayesian Theory is justified as an operational consistent decision-based measure of personal uncertainty. From a Bayesian point of view it has no sense to

think about *probability* if there is no decision maker, if there is no decision problem stated in terms of crisp actions. Bayesian rationality is about acts, no matter previous arguments justifying them. A Bayesian claims just to be consistent in our acts, because this is the only *observable* fact about our decision making process. Modeling personal acts is the only realistic goal from a Bayesian point of view. Bayesians will have no problems in agreeing with Klir [7] on the existence of different kinds of uncertainties. But a Bayesian would say it has no sense to consider uncertainties that have not been operationally defined, and only acts are operational. It is a waste of time to consider the existence of different kinds of uncertainties (or even more general models), since there is only one consistent and operational measure of personal uncertainty.

III. Decision Making versus Decision Processes.

The classical example of *tallness* as a fuzzy set may lead to some misunderstanding. A Bayesian approach where the degree of membership is understood as a likelihood indeed captures a particular uncertainty about *tallness*. A likelihood function seems fully appropriate in a conversation where the word *tall* is used in order to guess the particular *height* of John. In this case, the vague concept *tall* is used to carry out the lack of a desirable exact estimation about the crisp property *height*. There is nothing fuzzy about *height*.

But it may be the case that the required representation of the crisp attribute is not clear or it is too complex to be formalized, as for many composite concepts (see Saaty [14]). Being complex, an alternative fuzzy approach may be justified at least due to a low *operativeness* of the Bayesian approach. A key characteristic with *tallness* is that *height* can be represented in the real line. An analogous representation for more complex concepts may be in general far from being clear. Again, a pure Bayesian will be only interested in those acts associated to such complex concepts, whatever these complex concepts are.

Shafer's review [16] on Savage's classical work [15] provides, together with discussions, an excellent exposition of Bayesian paradoxes. For example, the Bayesian model assumes that every person has always well-defined complete transitive preferences, regardless any difficulty. Decision maker does not always define a complete order relation on the set of alternatives. Gambling with bets in search for fair prices may be not accepted after all (see also Shafer [18]). Moreover, the decision must be between clearly defined acts, so consequences can be checked (otherwise operationality is lost). Bets on fuzzy events are not so easy to accept. The bet "you win 10 dollars if the next man enters the room is tall; otherwise, you lose 10 dollars" does not give a rule of what to do if we do not agree on if the man just entering the room is *tall* or *not tall* (see [19]).

From our point of view, the main argument is that the concepts of *decision* and *act* should not be confused. Bayesians always talk about acts: they are particular and observable. But usually the observable act is a consequence of a previous inner decision process which is in fact the important human decision (which may be not observable in the Bayesian sense). For example, while walking on Berkeley streets one may decide to help a homeless sitting ahead on the sidewalk. Such a decision is not an act. Giving a particular amount of money is an act, and it depends on many circumstances (some of them are basically random, as the total amount of money each one of us has in our pockets and how it is distributed in bills and coins). Once one has decided to help that homeless, we are faced to the problem of choosing an act according to that decision, between those alternatives coming to our mind while still walking towards that homeless. Once well-defined acts have been chosen among all those at hand developing our previous decision about attitudes is where the Bayesian model works. Acts as alternatives use to appear as those (crisp) alternatives at the end of an inner hierarchical decision process. For example, once one has decided to help that homeless, we may think that "although sure he feels lonely, hungry and quite dirty, better give him spare change". Giving some spare change is neither a well-defined decision. In fact, we introduce one hand in our pocket, pick up at random a couple of coins and only then, when we give these two coins to that homeless, we have an *act*. Each step in the inner decision is about a personal attitude which is not well defined. A Bayesian would not take care about those previous non-observable decisions about attitudes (they are not acts, they are not crisp, they are not observable or testable).

Our objective may be to model an inner reasoning process, not just modeling observable acts. Our problem may be about decision processes. It is true that from acts we can try to guess how our mind worked out the final decision. But our inner decision process may not be making use at all of *acts* as pieces of the model, neither well defined consequences. In our homeless example, this is the case only once we are faced to the last step of what to do in order to help him: "how many and which ones from those coins in my pocket I'm gonna give him?"). In fact, we reach to the decision of helping that homeless after clearly ill-defined arguments. "When asked to make choices, they (people) look for arguments on which to base these choices", says Shafer [16]. Our decision making procedures are most of the time decision processes quite hierarchically structured (in fact, Saaty [14] claims that a fuzzy property is a hierarchical system of properties). Hopefully, this decision process involves at each step less and less abstract alternatives, so pure acts will appear at its final stage. "The preferences we construct depend on the questions we ask ourselves, and hence the selection of questions is an essential part of the construction" (Shafer [16]). We do not choose among all possible acts we can imagine. We build up decision processes by comparing *attitudes* (*policies*) which could be understood as poorly-defined families of acts, each one of these processes will at the end allow us to define a new set of feasible acts. It's a defuzzifying decision process leading to the construction of an *a priori* not known crisp set of feasible acts as the set of alternatives (see [16] for the importance of *constructive* methods). Choosing among these *attitudes* or *policies* means we are choosing among different models of actions, not particular acts, and in this context Bayesian Theory has problems in being adequately stated. Bayesian Theory will fit perfectly only into the last stage of such a decision process, when the problem has been already stated in terms of (crisp) acts.

Acts are the output of an inner decision making process. Decision making, if understood in a wide sense, contains decisions which are not only about acts. Human key decisions do not use to be acts (the important human decision was not giving two dimes to that homeless, but deciding to help him in some way). Modeling our acts is important, but our arguments should also deserve some effort. A pure Bayesian will claim that this second problem is not operational.

Language is an open system that evolves -like Science- searching for a better explanation of reality. No model -about language or Science- can fully explain reality. We can try to improve explanations by getting better and wider models, but if not based upon acts, again these models will be considered non operational by Bayesians.

Many Fuzzy theorists share with Frequentists the Platonic way they relate with reality. In fact, many arguments in Fuzzy Sets, as Frequentist Probability, come from the attempt to get mathematical models about the (perhaps subjective, but pre-existent) world. Such a model will most probably help us to make future decisions, but that's not the main issue to be taken into account in order to create a model. Better understanding of reality may be by itself the direct objective of our studies (though if we want to understand reality sure it is because I have some previous decision problem related with that piece of reality). Both Frequentist and many fuzzy models share a *descriptive* representation view, opposite to the *prescriptive* decision making approach of the Bayesian Theory (see comments of Dubois-Prade in Lavalette-Searman's paper [5]). The problem is again that a Bayesian claims that only *acts* are *operational*, so this is the only thing we should be modeling.

As pointed out by Shafer [16], probability provides us with a protocol for new information in statistical problems, "but in problems of everyday life, we constantly encounter unexpected information -information for which we have no protocol and no frequency experience." Again, Bayesian do not care about these arguments meanwhile they are non-observable. Since only individual acts are considered observable, Bayesians will have difficulties in modeling nothing except individual decision (act) making problems. It is not only that probability is personal and prescriptive what characterizes the Bayesian point of view, but the fact that the whole mathematical model is built from individual acts. It is not that Probability Theory provides a good model for uncertainty, but that individual acts -if consistent in the Bayesian sense- can be explained as if decision maker was playing lotteries.

Dempster's original work -Shafer [17] writes- was motivated "by the desire to obtain probability judgments based only on sample data, without dependence on prior subjective opinion". For example, non informativeness can be modeled by means of the pair of trivial upper and lower bounds, instead of any particular prior distribution. Probabilities, Shafer points out in [16], "should be constructed by examining evidence, not by examining one's attitude toward bets."

Taking the *path of realism* is defined by Roy [13] as "acknowledging that a certain number of objects, about which we can reason objectively, pre-exist 'out there' independently of any research carried out". This is the classical 'scientific' attitude, which is basically *descriptive* for discovering. Roy [13] is talking about the future of Decision Aid Sciences, and distinguishes such a *path of realism* from the *axiomatic path*, which basically looks for norms for prescribing (Roy notices that under this approach it is usually required some statement which uses the path of realism, as the existence of a complete order in the set of betting rates in the mind of a decision maker). Roy [13] claims that Decision-Aid Sciences should be mainly developed within a third path: the *path of constructivism*. This path acknowledges the difficulties in apprehending clearly and basically pursues to provide the basis for what he calls a *recommendation*. "The concepts, models, procedures and results are here seen as suitable tools for developing convictions and allowing them to evolve" (Roy [13]).

IV. Final Comment

A basic scheme of the Bayesian position should be understood in the following terms: in order to be useful, any theory has to be operational; and in order to be operational such a theory should be based upon observable facts; for a Bayesian, the only observable facts are personal acts, so these personal acts is the only thing we should be modeling; then, the only model that allows a consistent decision making procedure among acts is Probability. That does not mean that any probabilistic uncertainty exists: it just means that if an individual has a consistent behavior, his/her acts can be modeled as if he/she is playing lotteries. Any other approach to uncertainty will be non operational or will lead to non consistent decision making procedures.

The key claim in the Bayesian approach is that only (crisp) acts are observable. Listing advantages, features or properties is not helping to focus such a deep argument.

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Fuzzy Logic in Statistical Decision Theory*

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Abstract

In this paper we investigate applications of fuzzy theory to problems of decision-making. Primarily, we study the concept of fuzzy probability, the possibility of estimating such a probability, as well as the statistical testing of inexact hypotheses. Another problem that we will address is that of regression analysis with fuzzy data (see [17], [5], for the origin and previous development of this problem). Among the unifying methods that we plan to explore in this specific setting, are integrals with respect to nonadditive measures (i.e. Choquet integrals, [21]), set-valued probabilities [9], and optimization with inexact constraints [3].

Classical statistical techniques assume that both the data and the probability functions are represented by known numerical values. In particular, confidence estimation and testing of hypotheses, whether in a Bayesian framework or not, are based on the assumption that knowledge of data is precise.

There are, however, many instances (see, for example, [8], [6], [24]) when the available information is a mixture of randomness and fuzziness. A simple example of such a mixture is in random experiments when the observations are numerical, but the probabilities are fuzzy ([13], [25]).

In one of the models which we investigate we consider ordinary random variables but we assume that the probabilities are fuzzy-valued. The problems which we address will include: (1) Bayesian estimation of parameters when the prior information is fuzzy-valued; (2) estimation of a fuzzy probability; (3) testing of fuzzy hypotheses of the form "θ is F" (F being a fuzzy set); (4) fuzzy quantified rules and the aggregation of decision criteria.

Another problem that we will address is regression analysis with fuzzy data. This approach, pioneered in [17], was recently described within a least-squares framework in [5]. However, the problem (even for interval-valued data) is much more complex, and we will describe a better fit for the data using special linear operators in $\mathbf{R}^n$ which leave a cone invariant.

1 Introduction

Consider the following simple examples:

(a) A student in a class is chosen at random. What is the probability that the student is *tall*?

(b) It is known that the score on a particular test follows the normal distribution with mean 65 and standard deviation 2.4. For a student chosen at random, what is the probability that he/she had a *very high* score on that test?

All these examples have in common a mixture of randomness and fuzziness. The experiment is random, giving rise to a (real-valued) random variable, X. However, the fuzziness is present in terms of the range of the random variable. More specifically, we want to calculate a probability of the form $P(X \text{ is } M)$ where M is a fuzzy set. To do this, we have to address the basic problem of a probability of a fuzzy event. The first to recognize the importance of this problem was Zadeh [22] who defined the probability of a fuzzy event as a real, (non-fuzzy) number between 0 and 1. To review this concept briefly, let $(\Omega, \mathcal{P}, P)$ be a probability space; Ω is the sample space of all outcomes of some random experiment; $\mathcal{P}$ is a σ-algebra of subsets of Ω (called events), and $P : \mathcal{P} \to [0,1]$ is a probability measure in the usual sense. A fuzzy event in Ω is a fuzzy subset A of Ω, given by its membership function $\mu_A : \Omega \to [0,1]$.

Then the probability of a fuzzy event A [22] is defined as

$$(1) \qquad P(A) = E(\mu_A) = \int_\Omega \mu_A dP$$

where $E()$ stands for the expected value.

Example 1: If Ω is a finite set with n elements and if P is the uniform measure assigning probability

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