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Bulk viscosity in a cold CFL superfluid

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Abstract. We compute one of the bulk viscosity coefficients of cold color–flavor locked (CFL) quark matter in the temperature regime where the contribution of mesons, quarks and gluons to transport phenomena is Boltzmann suppressed. In that regime dissipation occurs due to collisions of superfluid phonons, the Goldstone modes associated to the spontaneous breaking of baryon symmetry. We first review the hydrodynamics of relativistic superfluids, and recall that there are at least three bulk viscosity coefficients in these systems. We then compute the bulk viscosity coefficient associated to the normal fluid component of the superfluid. In our analysis we use Son's effective field theory for the superfluid phonon, amended to include scale breaking effects proportional to the square of the strange quark mass $m_{\rm s}$. We compute the bulk viscosity at leading order in the scale breaking parameter, and find that it is dominated by collinear splitting and joining processes. The resulting transport coefficient is $\zeta = 0.011\ m_{\rm s}^4/T$, growing at low temperature T until the phonon fluid description stops making sense. Our results are relevant for studying the rotational properties of a compact star formed by CFL quark matter.

Keywords: neutron stars, gravity

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Contents

1. Introduction

In this paper we present the computation of one of the bulk viscosity coefficients of color–flavor locked (CFL) quark matter [1] at low temperature. This work represents a follow up of [2], where the shear viscosity in the CFL phase was computed in the cold regime where the contribution of all the gapped particles (mesons, quarks and gluons) is Boltzmann suppressed.

The density reached in the core of neutrons stars might be so high that all the hadrons could be melted into their fundamental constituents. This consideration has motivated the studies of Quantum Chromodynamics (QCD) at high baryonic density and low temperature [3]. At least in the asymptotic high density regime, when the QCD coupling constant is small, reliable theoretical predictions for the behavior of quark matter can be formulated. Further, it has been known for long time that cold dense quark matter should exhibit the phenomenon of color superconductivity. In order to connect theoretical predictions with possible astrophysical signatures of quark matter [4], it is important to have a precise knowledge of both the equation of state of quark matter and also of all transport coefficients, which are very sensitive to the presence of superconductivity in the system.

It has been established that the viscosities put stringent tests on astrophysical models for very rapidly rotating stars, such as for millisecond pulsars. This is based on the existence of r(otational)-mode instabilities in all relativistic rotating stars [5], which are only suppressed by sufficiently large viscosities. So the viscosities allow us to discard unrealistic models for millisecond pulsars. There are many different color superconducting phases, and their occurrence depends on both the values of the baryonic chemical potential and of the different quark masses. At present, there are several computations of viscosity coefficients in the different quark matter phases [6]–[12]. All of them have the motivation of studying the development of the r-modes of a compact star made of, partly or entirely, unconfined quark matter.

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Here we will only be concerned about the CFL phase, which is the preferred phase in the presence of three light quark flavors. The CFL phase is special in many ways, as its long distance physics is very similar to the corresponding one of QCD *in vacuo* [1, 13]. Here we want to stress that it has very peculiar hydrodynamics. In the CFL case the baryon symmetry is spontaneously broken, and thus CFL quark matter becomes a superfluid of the same sort as those found in condensed matter systems, such as in Bose–Einstein condensates. Landau developed the hydrodynamical description of these non-relativistic superfluids, proposing his famous two-fluid model [14, 15]. The hydrodynamics associated to relativistic superfluids has been much less studied, although the two-fluid model has been generalized to the relativistic domain [16]–[19]. We believe that the CFL superfluid may represent one specific and beautiful example where the sophisticated relativistic superfluid hydrodynamics could be derived from first principles, at least in the asymptotic high density domain. One of the peculiarities of these superfluids is that they have more viscosities than a normal fluid, as one can define more than one hydrodynamical velocity.

In this paper we compute the bulk viscosity coefficient associated to the normal fluid component of the cold CFL superfluid. Transport coefficients are very sensitive to the temperature T of the system. In the regime where T is smaller than all the energy gaps of all the quasiparticles (mesons, quarks and gluons), transport coefficients in the CFL phase are dominated by the collisions of the superfluid phonons [2, 20], and these will be the relevant modes in our study. In our analysis we use Son's effective field theory for the superfluid phonon [21]. However, Son's theory is scale invariant, and leads to a vanishing bulk viscosity, as this transport coefficient measures the dissipation after a volume compression or expansion of the system. We then consider scale breaking effects due to a non-vanishing value of the strange quark mass. The inclusion of quark masses, that represent an explicit chiral symmetry breaking effect in the QCD Lagrangian, makes the octet of (pseudo) Goldstone bosons of the CFL phase (the pions, the kaons, the eta) massive. Because a quark mass term in the Lagrangian respects the baryon symmetry, the superfluid phonon remains massless, although quark mass effects still affect its dynamics, as we will see. We compute the bulk viscosity at leading order in this scale breaking parameter, and find that it is dominated by collinear splitting and joining processes. Surprisingly, the computation shares many technical similarities with that of the bulk viscosity in the hot, weakly coupled, phase of QCD at zero chemical potential [22], as we will later point out.

Let us stress that at higher temperatures other quasiparticle modes might be relevant for bulk viscosity as well. In [11] the bulk viscosity due to kaons in the CFL phase has been computed. Allowing for flavor changing processes, mediated by the electroweak interactions, the bulk viscosity has been computed assuming that the relevant processes are those of a neutral kaon decaying into two superfluid phonons, and to $K^\pm \leftrightarrow e^\pm + \nu$. However, as found in that reference, for temperatures below the energy gap associated to the kaon, δm, the kaon contribution to bulk viscosity is exponentially suppressed $\sim e^{-\delta m/T}$, as naturally expected. There is an additional uncertainty of at what temperatures this suppression is effective, as the value of the kaon masses computed in the literature can only be trusted in the asymptotic high density limit. They are believed to be in the range of a few MeV, or slightly higher [23]–[28].

This paper is structured as follows. In section 2 we recall the hydrodynamical equations of a relativistic superfluid. Section 3 is devoted to reviewing Son's effective field theory for the superfluid phonon. With the same effective field theory, one can easily also derive the dispersion law for the phonon in a moving superfluid, introducing the concept of acoustic or sonic metric, which also allows us to identify these phonons with the sound waves of the superfluid. In section 4 we see how scale breaking effects could be included in Son's Lagrangian. The explicit computation of the bulk viscosity is given in section 4. After identifying the leading collisional processes relevant for this transport coefficient, we write down the Boltzmann equation for the phonon, and linearize it in the small deviations around equilibrium in section 5.1. The collision term is explicitly written down in section 5.2, and the numerical results of our computation are displayed in section 5.3. We conclude with a discussion of our results in section 6. We will use throughout natural units $\hbar = c = k_{\rm B} = 1$ and the metric conventions $(1, -1, -1, -1)$.

2. Hydrodynamics in relativistic superfluids

In this section we present a quick review of the hydrodynamical equations for a relativistic superfluid. These represent the natural relativistic generalization of Landau's two-fluid model of superfluid (non-relativistic) dynamics [14, 15]. There are different formulations of the hydrodynamics of a relativistic superfluid [16]–[19], they all differ in the choice of the hydrodynamical variables used to describe the fluids.

The hydrodynamical equations in the superfluid take the form of conservation laws, as in a normal fluid. If n^ρ is the particle current (in our case, the baryon current) these are

$$\partial_\rho n^\rho = 0, \tag{1}$$

and the energy–momentum conservation law

$$\partial_\rho T^{\rho\sigma} = 0. \tag{2}$$

In the absence of dissipation, the entropy current is conserved, and thus one further has

$$\partial_\rho s^\rho = 0. \tag{3}$$

In the superfluid, the gradient of the phase of the condensate allows to define the four-vector

$$\mu_\rho = \partial_\rho \varphi. \tag{4}$$

The approach of Carter and Khalatnikov [17] defines all the hydrodynamical equations based on expressing both the particle current and the energy–momentum tensor in terms of μ_ρ (or φ) and s_ρ. In particular, in [18] it is shown that the energy–momentum tensor can be written as

$$T^{\rho\sigma} = A\mu^\rho\mu^\sigma + Bs^\rho s^\sigma - Pg^{\rho\sigma}, \tag{5}$$

where P is the generalized pressure of the system. The coefficients A and B can be obtained with the knowledge of P. Similarly, n^ρ can be expressed in terms of both μ^ρ and s^ρ.

Son formulated a different description of the hydrodynamics of the relativistic superfluids in [19]. After a non-trivial mapping of his variables, their equations can be converted to the Carter and Khalatnikov form [29].

In the zero temperature limit, when the pressure is only a function of the chemical potential, the entropy current vanishes. It is only in this case when the energy–momentum tensor takes the form of that of an ideal fluid [18]. If we define the velocity vector

$$u^\rho = \frac{\mu^\rho}{\mu} \tag{6}$$

such that is properly normalized, $u^\rho u_\rho = 1$, then in the cold $T \to 0$ limit one has [18]

$$T^{\rho\sigma} = (n\mu) u^\rho u^\sigma - P g^{\rho\sigma} = (\epsilon + P)\, u^\rho u^\sigma - P g^{\rho\sigma}, \tag{7}$$

where ϵ is the energy density of the system, and we have used the zero temperature relation $n\mu = \epsilon + P$. In this case the associated hydrodynamical equations are the same as in an ideal fluid. The entropy strictly vanishes, and thus there is no dissipation.

At finite temperature, the entropy does not vanish, and dissipational processes are responsible for entropy production. Dissipative relativistic superfluid hydrodynamical equations have only been derived, to the best of our knowledge, in [16], although there is a vast literature on the subject for non-relativistic superfluids [14, 15]. In [16], and after imposing that deviations from the dissipationless particle current and energy–momentum are expressed in terms of linear gradients of the basic hydrodynamical variables, and that entropy production is positive-definite, it was found that more kinetic coefficients that in a normal fluid can be defined. We leave for a future project a much more detailed discussion on all the possible transport coefficients that can be defined in the CFL superfluid. We simply note here that in a non-relativistic superfluid four viscosity coefficients can be defined [14, 15], and thus, at a minimum, the same number of viscosity coefficients in the relativistic domain exist.

In this paper we compute the bulk viscosity coefficient associated to the normal fluid. For that purpose, and taking into account that the bulk viscosity is a Lorentz scalar, we will work in the superfluid rest frame, as this simplifies the computation enormously. More specifically, we will compute the dissipative term in the energy–momentum tensor that goes as

$$T^{ij}_{\mathrm{d}} = -\zeta\, \delta^{ij}\, \boldsymbol{\nabla} \cdot \mathbf{V}, \tag{8}$$

where $\mathbf{V}$ is the velocity of the normal fluid in the superfluid rest frame.

3. The cold CFL superfluid and the sonic metric

In this section we review the effective field theory of the superfluid phonon constructed by Son [21]. Further, we introduce the concept of the sonic or acoustic metric, which is rather convenient in order to describe the dynamics of the phonon moving in the background of the superfluid. It also allows us to give the interpretation of the superfluid phonon as a sound wave.

In a non-relativistic superfluid, gravity analogs for the description of the low energy collective modes or superfluid phonons have been developed (see [30, 31] and references therein). In such an approach, one treats the superfluid as a gravitational background, in which the quasiparticles, composing the normal fluid, propagate. We will use the same analogy here.

Son showed that the effective field theory for the only truly Goldstone boson of the CFL phase can be constructed from the equation of state (EOS) of normal quark matter [21]. If φ is the phase of the condensate, and one defines $D_\rho\varphi \equiv \partial_\rho\varphi - (\mu, \mathbf{0})$, then the effective Lagrangian for φ is expressed as

$$\mathcal{L}_{\rm eff}[D_\rho\varphi] = P[\mu = (D_\rho\varphi D^\rho\varphi)^{1/2}], \tag{9}$$

where P is the pressure of the system at zero temperature.

At asymptotic large densities the EOS of CFL quark matter reads

$$P[\mu] = \frac{3}{4\pi^2}\mu^4, \tag{10}$$

where μ is the quark chemical potential. At very high μ, when the coupling constant is small $g(\mu) \ll 1$, the effects of interactions and the effects of Cooper pairing are subleading and neglected in equation (10). Also, one assumes that the quark masses give a subleading effect, as $m_q \ll \mu$. From equation (10) Son obtained the effective Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{3}{4\pi^2}\left[(\partial_0\varphi - \mu)^2 - (\partial_i\varphi)^2\right]^2. \tag{11}$$

There is an interesting interpretation of the equations of motion associated to φ. The classical equations of motion derived from the above Lagrangian can be rewritten as the hydrodynamical conservation law of a current representing baryon number flow,

$$\partial_\nu(n_0\bar{u}^\nu) = 0, \tag{12}$$

where $n_0 = \frac{{\rm d}P}{{\rm d}\mu}|_{\mu=\mu_0}$ is interpreted as the baryon density [21]. Son defined the superfluid velocity $\bar{u}^\rho$ as being proportional to the gradient of the condensate phase, [14],

$$\bar{u}_\rho = -\frac{D_\rho\bar{\varphi}}{\mu_0}, \tag{13}$$

where $\mu_0 = (D_\rho\bar{\varphi}D^\rho\bar{\varphi})^{1/2}$. It only differs from the choice of the last section by an irrelevant constant, although the hydrodynamics is completely analogous. The energy–momentum tensor associated to the theory described in equation (11) can also be written in terms of the velocity defined in equation (13) and Noether's energy-density ϵ,

$$T^{\rho\sigma} = (\epsilon + P)u^\rho u^\sigma - g^{\rho\sigma}P. \tag{14}$$

It is conserved and traceless:

$$\partial_\rho T^{\rho\sigma} = 0, \qquad T^\rho_\rho = 0. \tag{15}$$

Equations (12) and (15) are the hydrodynamic equations for the relativistic superfluid [21]. They need modifications at finite temperature, as explained in the previous section. At low temperatures, the superfluid phonons are thermally excited (and compose the normal fluid component), which are responsible for the entropy current in the system. Other particles may also conform an additional component to the normal fluid, but at low temperatures, as discussed in the introduction, their contribution to the hydrodynamics is Boltzmann suppressed.

Let us also mention that Son's Lagrangian yields the effective field theory of the phonons moving in the background of the superfluid. The phonon is a Goldstone boson, given also by the phase of the condensate. When the superfluid is at rest, their interactions are given in equation (11). To find the phonon dispersion relation in a moving superfluid we will simply consider the quantum fluctuations around the classical solution of equation (11):

$$\varphi(x) = \bar{\varphi}(x) + \phi(x). \tag{16}$$

The action associated to Son's Lagrangian

$$S[\varphi] = \int \mathrm{d}^4x\, \mathcal{L}_{\rm eff}[\partial\varphi] \tag{17}$$

is then expanded around the classical solution

$$S[\varphi] = S[\bar{\varphi}] + \frac{1}{2}\int \mathrm{d}^4x \left\{ \frac{\delta^2 \mathcal{L}_{\rm eff}}{\delta(\partial_\mu\varphi)\delta(\partial_\nu\varphi)} \right\} \partial_\mu\, \phi\partial_\nu\phi + \cdots. \tag{18}$$

The equation of motion of the linearized fluctuation—here the superfluid phonon—can be written as that of a boson moving in a non-trivial gravity background:

$$\partial_\mu \left(\sqrt{-\mathcal{G}}\, \mathcal{G}^{\mu\nu}\partial_\nu\phi \right) = 0, \tag{19}$$

where, in this case, we identified

$$\sqrt{-\mathcal{G}}\, \mathcal{G}^{\mu\nu} = \left. \frac{\delta^2 \mathcal{L}_{\rm eff}}{\delta(\partial_\mu\varphi)\delta(\partial_\nu\varphi)} \right|_{\bar{\varphi}} = \frac{3\mu^2}{2\pi^2} \left\{ g^{\mu\nu} + \left(\frac{1}{c_{\rm s}^2} - 1 \right) \bar{u}^\mu \bar{u}^\nu \right\}, \tag{20}$$

where $c_{\rm s} = 1/\sqrt{3}$ is the speed of sound in the ultrarelativistic system. Thus, from Son's Lagrangian, we have derived the so-called sonic or acoustic metric tensor $\mathcal{G}^{\mu\nu}$ [18, 30]. One then finds the phonon dispersion relation in the moving superfluid as solutions of the equation

$$\mathcal{G}^{\mu\nu}k_\mu k_\nu = 0, \tag{21}$$

where $k_\mu = (E, \mathbf{k})$.

Let us note that in an ideal fluid the fluctuations of the pressure, δp, obey the sound wave equation (see, e.g., equation (23) of [32]), which in Fourier space reads

$$\frac{1}{\bar{u}^\mu k_\mu} \left\{ \left(\frac{1}{c_{\rm s}^2} - 1 \right) (\bar{u}^\mu k_\mu)^2 + k^2 \right\} \delta p = 0. \tag{22}$$

Simply by defining

$$\delta p = \frac{3\mu^2}{2\pi^2} (\bar{u}^\mu \partial_\mu)\, \phi \tag{23}$$

we derive the same dispersion equation (21). This is naturally so, as the superfluid phonons describe the sound waves associated to the superfluid component of the fluid. Let us stress here that in the relativistic superfluid there are two (first and second) sound speeds, associated to the fact that there are two different fluid components [18].

In the superfluid rest frame, that is, where $\bar{u}^\mu = (1, 0, 0, 0)$, the phonon dispersion relation simplifies to the form

$$E_k = c_{\rm s} k, \tag{24}$$

as can be easily checked. This is the frame in which we will perform our computations. However, we want to stress the fact that this would not be correct in order to compute other transport coefficients in the system that involve the superfluid velocity $\bar{u}^\mu$. In those cases, the gravity analogs described here become a very efficient tool, which will be exploited in future references. Although sonic metrics are usually employed to construct simple laboratory analog models of Einsteinian gravity [33], we think the method also shows promise in reverse, for the involved features of relativistic superfluids might be conceptually understood in the framework of gravity analogs.

4. Effective field theory for the superfluid phonons including scale breaking effects

In [2], the effective field theory Lagrangian (11) was used to compute the shear viscosity in the phonon fluid of cold CFL quark matter. For the computation of the bulk viscosity one has to introduce corrections to that effective field theory. As signaled by the vanishing of the trace of the energy–momentum tensor, equation (15), this is an scale invariant theory. Bulk viscosity is a transport coefficient that measures the dissipation after a volume compression or expansion, and it vanishes exactly for a scale invariant relativistic theory. Thus, in this section we will look for corrections to equation (11) that introduce scale breaking effects.

The quantum scale anomaly breaks the conformal symmetry in the system, introducing, through dimensional transmutation, the quantum scale $\Lambda_{\rm QCD}$. One could then compute g-corrections to Son's effective field theory, arising as g-corrections to the pressure of quark matter. These corrections would introduce new terms in the superfluid phonon Lagrangian proportional to the QCD beta function. In the very high μ limit, when $g(\mu) \ll 1$, we expect this to be a rather negligible effect.

The inclusion of quark mass effects in the system also breaks scale invariance. In CFL quark matter, the quark masses represent an explicit chiral symmetry breaking effect, which gives masses to the associated (pseudo) Goldstone bosons (the pions, kaons and eta). A quark mass term in the QCD Lagrangian respects baryon symmetry, and thus it does not make the superfluid phonon massive; however, it still affects its dynamics, as we show below.

Since all three light quarks participate in the CFL phase, the largest effect comes from the strange quark mass $m_{\rm s}$, and we ignore the masses of the up and down quarks, as $m_{\rm u}, m_{\rm d} \ll m_{\rm s}$. Further, we will consider that $m_{\rm s}^2 < 2\Delta\mu$ [3], which is the threshold value under which the CFL phase is stable. We will always work in a leading order expansion in $m_{\rm s}^2/\mu^2$. After imposing the constraints of electrical neutrality and beta equilibrium of quark matter, the first correction at order $m_{\rm s}^2/\mu^2$ to the pressure reads [34]

$$P[\mu] = \frac{3}{4\pi^2}\left(\mu^4 - \mu^2 m_{\rm s}^2\right). \tag{25}$$

To this order, first in the $m_{\rm s}^2/\mu^2$ expansion, and since isospin breaking effects happen to be of the same order, one must specify that μ refers to precisely $\mu_n/3$ in terms of the baryon chemical potential. Let us stress here that, as in Son's theory, equation (10), we neglect both the effects of interactions and of Cooper pairing, assuming the asymptotic high density, and thus the weak coupling domain. The expansion here is aimed at considering the most relevant scale breaking effect that corrects Son's Lagrangian.

Following the same procedure advocated by Son, with the knowledge of the pressure we get the effective field theory for the phonons or sound waves in the system

$$\mathcal{L}_{\rm eff}^{m} = \frac{3}{4\pi^2}\left[\left((\partial_0\varphi - \mu)^2 - (\partial_i\varphi)^2\right)^2 - m_{\rm s}^2\left((\partial_0\varphi - \mu)^2 - (\partial_i\varphi)^2\right)\right]. \tag{26}$$

We now rescale the phonon field

$$\tilde{\varphi} = \frac{3\mu}{\pi}\sqrt{1 - \frac{m_{\rm s}^2}{6\mu^2}}\,\varphi \tag{27}$$

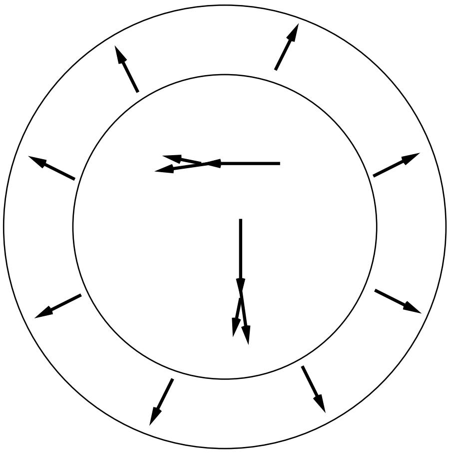

Figure 1. Under spherically symmetric, radially non-uniform rarefaction (compression) of a gas not invariant under dilatations, the pressure diminishes below (increases above) its equilibrium value. For the superfluid phonon gas, equilibrium is restored by phonon collinear splitting (joining).

to normalize the kinetic term in accordance with the Lehmann–Symanzik–Zimmermann formula. Then the Lagrangian for the rescaled field reads

$$\mathcal{L}_{\rm eff} = \frac{1}{2}(\partial_0\tilde{\varphi})^2 - \frac{c_{\rm s}^2}{2}(\partial_i\tilde{\varphi})^2 - g_3\partial_0\tilde{\varphi}(\partial_\mu\tilde{\varphi}\partial^\mu\tilde{\varphi}) + g_4(\partial_\mu\tilde{\varphi}\partial^\mu\tilde{\varphi})^4, \tag{28}$$

where, to order $m_{\rm s}^2/\mu^2$, we find

$$c_{\rm s}^2 = \frac{1}{3}\left(1 - \frac{m_{\rm s}^2}{3\mu^2}\right), \tag{29}$$

$$g_3 = \frac{\pi}{9\mu^2}\left(1 + \frac{m_{\rm s}^2}{4\mu^2}\right), \qquad g_4 = \frac{\pi^4}{108\mu^4}\left(1 + \frac{m_{\rm s}^2}{3\mu^2}\right). \tag{30}$$

Thus, the leading order effect of the strange quark mass in the phonon Lagrangian is to modify the velocity of the phonon, that obviously equals the speed of sound in the system without conformal symmetry, and a finite renormalization of the cubic and quartic self-couplings.

5. Transport theory in the phonon fluid

We now turn to the microscopic description of the bulk viscosity at low temperature. The dominant process relevant for the bulk viscosity, shown in figure 1, is phonon collinear splitting or joining processes $1 \leftrightarrow 2$. Large angle $2 \leftrightarrow 2$ scatterings are suppressed, as compared to collinear splitting, by powers of $1/\mu^2$, as found in [2]. Because we are considering the regime $T \ll \mu$, we can safely neglect those collisions. Further, we do not consider small angle $2 \leftrightarrow 2$ collisions, which are collinearly enhanced [2]. Considering simultaneously the $1 \leftrightarrow 2$ processes and small angle $2 \leftrightarrow 2$ collisions would mean to incur in a wrong double-counting.

It is remarkable that the computation in the asymptotic large density and low temperature CFL phase has many points in common with the same computation in the very hot, weakly coupled phase of QCD at vanishing chemical potential. In the hot phase of QCD, bulk viscosity is dominated by both effective collinear splitting processes $1 \leftrightarrow 2$, as well as by $2 \leftrightarrow 2$ collisions [22]. Fortunately, in the CFL phase the computation is simpler, as the last processes are certainly suppressed. However, we will find convenient to follow the same technical strategy as that in [22], and tackle different subtle points in the computation in the same way as in that reference, which we will closely follow.

There is a technical remark different from the QCD case that we would like to point out here. Collinear splitting is closed by a convex dispersion relation when one considers corrections to the phonon dispersion relation at order k^2/Δ^2 [35], where Δ is the superconducting gap. If one considers those corrections, one should then study other number changing collisions, such as a $2 \leftrightarrow 3$ scattering for the computation of the bulk viscosity. In these $2 \leftrightarrow 3$ processes, one may treat one of the particles as a spectator whose only role is to restore energy–momentum conservation in the splitting vertex. We will ignore this subtlety here and operate as if the dispersion relation was exactly linear, kinematically allowing for collinear splitting and rejoining. We expect that considering those processes would simply allow us to get corrections of order T^2/Δ^2 to our leading result.

5.1. The Boltzmann equation and bulk viscosity

In the superfluid rest frame the out of equilibrium phonon distribution function evolves according to the Boltzmann equation

$$\frac{\partial f_p}{\partial t} + \mathbf{v_p} \cdot \boldsymbol{\nabla}_x f_p = -C[f_p], \tag{31}$$

where $\mathbf{v_p} = \boldsymbol{\nabla}_p E_p$, and $C[f]$ is the collision integral. We have also introduced the shorthand notation $f_p = f(x, \mathbf{p})$ that we will use in what follows.

We will consider small deviations from equilibrium:

$$f_p = f_p^{\rm eq} + f_p^1 + \cdots. \tag{32}$$

In the phonon rest frame,

$$f_p^{\rm eq} = f_p^0 = \frac{1}{{\rm e}^{\beta E_p} - 1}, \tag{33}$$

where $\beta = 1/T$.

The Boltzmann equation is linearized in the departures of equilibrium. One has to keep in mind that the collision term evaluated with the equilibrium function vanishes by detailed balance. As in [22], we cast the left-hand side of the Boltzmann equation with separated variables as

$$X(x)\beta f_p^0(1 + f_p^0)q(p) \tag{34}$$

that parameterizes the advective derivative of f by defining $q(p)$. In order to do so, one needs some thermodynamical relations. In the superfluid rest frame, those are $\epsilon + P = T{\rm d}P/{\rm d}T$, exactly as in the hot quark–gluon plasma at zero chemical potential. The explicit calculation of this advective term in [22] yields

$$q(p) = \frac{\mathbf{p} \cdot \mathbf{v}_p}{3} - c_{\rm s}^2 \frac{\partial(\beta E_p)}{\partial \beta} = \left(\frac{1}{3} - c_{\rm s}^2\right) E_p, \tag{35}$$

and for a departure of equilibrium that is a uniform compression,

$$X(x) = \boldsymbol{\nabla} \cdot \mathbf{V}, \tag{36}$$

as relevant for the bulk viscosity.

Equation (35) reflects the fact that in a relativistic scale invariant theory, where $c_{\mathrm{s}}^2 = 1/3$, $q(p) = 0$, and thus the bulk viscosity vanishes. Thus, we allow for scale breaking effects, as anticipated in the previous section. A non-vanishing strange quark mass, $m_{\mathrm{s}} \neq 0$, introduces a correction to the speed of sound, as found in equation (29). In this case we obtain

$$q(p) = \frac{p^2}{9E_p}\frac{m_{\mathrm{s}}^2}{\mu^2}, \tag{37}$$

proportional to our scale breaking parameter m_{s}^2/μ^2.

The next step in the analysis is to project the Boltzmann equation into weak (integrated over p) form amenable to variational treatment. To simplify notation it is convenient to introduce a scalar product

$$\langle \, | \, \rangle = \beta^3 \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}. \tag{38}$$

Multiplying the Boltzmann equation by $-T^2$, we obtain

$$-T f_p^0(1 + f_p^0)\frac{p^2}{4E_p}\frac{m_{\mathrm{s}}^2}{\mu^2}X(x) = -T^2 C[f_p^0 + f_p^1], \tag{39}$$

and we define, as in [22], a function for the left-hand side source S, without either the $X(x)$ factor (that will cancel left and right in the Boltzmann equation) or the conformal symmetry breaking factor, $(m_{\mathrm{s}}/\mu)^2$,

$$S(p) = -T f_p^0(1 + f_p^0)\frac{p^2}{4E_p}. \tag{40}$$

The projection of this equation over a complete orthonormal basis of functions ψ_n is then, formally,

$$X(x)\frac{m_{\mathrm{s}}^2}{\mu^2}\langle \psi_n | S \rangle = \langle \psi_n | C[f_p^0 + f_p^1] \rangle. \tag{41}$$

And we further introduce a dimensionless variable (henceforth, the bar notation denotes an adimensional function of momenta over temperature)

$$S(p) = -T^2 \bar{S}(p/T).$$

The bulk viscosity is given as the matrix element

$$\zeta = \frac{m_{\mathrm{s}}^4}{\mu^4}\langle S | \left(\frac{\delta C}{\delta f}\right)^{-1} | S \rangle \tag{42}$$

with the differential matrix of the collision operator

$$\left(\frac{\delta C}{\delta f}\right)_{mn} = \langle \psi_m | \frac{\delta C[f]}{\delta f} | \psi_n \rangle. \tag{43}$$

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5.2. The collision term

Bulk viscosity involves the relaxation of a momentum gradient along the same direction of the momentum. Within Son's effective theory, the lowest order effect that achieves this is the collinear splitting induced by the cubic term. Because our computation is done at leading order in the scale breaking parameter, $m_{\rm s}^2/\mu^2$, and this has already taken into account in equation (35), it is enough to keep the scattering matrices as arising in the scale invariant theory. Thus, in this section, we take the value $c_{\rm s} = 1/\sqrt{3}$. Let us insist again on the fact that collisions involving two bosons in the initial state and two bosons in the final state are suppressed by a further power of μ^2 at the amplitude level, and thus we neglect them.

We will need therefore the amplitudes for a boson of momentum p to split into two bosons of momenta p', k' and the amplitude for a boson of momentum p' to split and give back in the final state the p boson and a k' boson. From Son's Lagrangian density, these are found to be

$$\mathcal{M}(p;p',k') = \frac{-\mathrm{i}2\pi}{9\mu^2}(p^0(p'\cdot k') + p'^0(p\cdot k') + k'^0(p\cdot p')), \tag{44}$$

or, employing momentum conservation and the linear dispersion relation $p^0 = c_{\rm s}|p|$,

$$|\mathcal{M}(p;p',k')|^2 = \frac{4\pi^2}{81\mu^4}c_{\rm s}^2|p|^2|k'|^2 4x^2(|p|-|k'|)^2, \tag{45}$$

where $x = \hat{p}\cdot\hat{k}'$. Similarly, one finds

$$|\mathcal{M}(p';p,k')|^2 = \frac{4\pi^2}{81\mu^4}c_{\rm s}^2|p|^2|k'|^2 4x^2(|p|+|k'|)^2. \tag{46}$$

The form of the collinear splitting collision terms that enter into the Boltzmann equation can be read off [36]:

$$\begin{aligned} C^{1\to2}[f_p] &= C_a^{1\to2}[f_p] + C_b^{1\to2}[f_p] \\ &= \frac{1}{4E_p}\left[\int\right]_{p'k'}(2\pi)^4\delta^{(4)}(p-p'-k')\left(f_p(1+f_{p'})(1+f_{k'}) - f_{p'}f_{k'}(1+f_p)\right) \\ &\quad + \frac{1}{2E_p}\left[\int\right]_{p'k'}(2\pi)^4\delta^{(4)}(p'-p-k')(f_{p'}(1+f_p)(1+f_{k'}) - f_pf_{k'}(1+f_{p'})), \end{aligned} \tag{47}$$

where we introduced the shorthand notation

$$\left[\int\right]_{p'k'} \equiv \int\int \frac{\mathrm{d}^3\mathbf{k}'}{2E_{k'}(2\pi)^3}\frac{\mathrm{d}^3\mathbf{p}'}{2E_{p'}(2\pi)^3}|\mathcal{M}(p;p',k')|^2.$$

Let us reduce equation (47) to a more tractable form. The idea is to split the momentum conservation delta into an energy part, a longitudinal part (defined along p), and a transverse part. The best way to organize the calculation is to introduce a collinear splitting function

$$\begin{aligned} \gamma(p;k',p') = \int &\frac{\mathrm{d}^2k'_\perp\,\mathrm{d}^2p'_\perp}{(2\pi)^3 4c_{\rm s}^3|p'||k'|}\frac{|p|}{2} \\ &\times |\mathcal{M}(p;p',k')|^2\delta^{(2)}(p_\perp - p'_\perp - k'_\perp)\delta(E_p - E_{p'} - E_{k'}), \end{aligned} \tag{48}$$

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so that the first collision term in equation (47) becomes

$$C_a^{1\to 2}[f_p] = \frac{2\pi}{2|p|^2}\int_0^\infty \int_0^\infty \mathrm{d}|k'|\,\mathrm{d}|p'|\gamma(p;p',k')\delta(|p|-p'_L-k'_L)$$
$$\times\ (f_p(1+f_{p'})(1+f_{k'}) - f_{p'}f_{k'}(1+f_p))\,. \tag{49}$$

Now, energy conservation in equation (48) forces the transverse momentum to vanish:

$$\delta(c_\mathrm{s}(|p|-|p'|-|k'|)) = \frac{\delta(|k'_\perp|)}{c_\mathrm{s}\left(k'_\perp/\sqrt{p'^2_L+k'^2_\perp}+k'_\perp/\sqrt{k'^2_L+k'^2_\perp}\right)}. \tag{50}$$

Thus

$$\gamma(p;p'=p-k',k') = \frac{1}{(2\pi)^2 8c_\mathrm{s}^4}\left(\frac{16\pi^2c_\mathrm{s}^2}{81\mu^4}\right)|p|^2|k'|^2(|p|-|k'|)^2. \tag{51}$$

Finally, one can express the collision term as

$$C_a^{1\to 2} = \frac{2\pi}{2|p|^2}\int_0^\infty \int_0^\infty \mathrm{d}|k'|\,\mathrm{d}|p'|\gamma(p;p',k')\delta(|p|-|p'|-|k'|)$$
$$\times\ (f_p(1+f_{p'})(1+f_{k'}) - f_{p'}f_{k'}(1+f_p))\,, \tag{52}$$

with

$$\gamma(p;p'=p-k',k') = \left(\frac{1}{81\mu^4\cdot 2c_\mathrm{s}^2}\right)|p|^2|k'|^2|p'|^2. \tag{53}$$

We can analogously reduce the second term of equation (47), that amounts essentially to the exchange $p\to p'$ with respect to the $C_a^{1\to 2}$ piece, as the splitting function is totally symmetric in its three arguments. Thus, we will not write here the analogous equation for $C_b^{1\to 2}$.

We linearize the collision term in the first order in the deviation from equilibrium, which we parameterize as

$$f_p^1 = \frac{X(x)}{T}f_p^0(1+f_p^0)\frac{\chi_p}{T}. \tag{54}$$

The last line in equation (52) containing the distribution functions becomes

$$F(p;p',k') \equiv \frac{X(x)}{T}\frac{\chi_{p'}-\chi_{k'}-\chi_p}{T}f_p^0(1+f_{p'}^0)(1+f_{k'}^0). \tag{55}$$

We then expand the deviation from equilibrium in terms of the function basis $\chi(p)/T = \sum_n \chi_n\psi_n(p/T)$. Then the projected, linearized collision operator turns out to be

$$\langle\psi_m|\frac{\delta C[f]}{\delta f}|\psi_n\rangle = 4\pi^2\int\int\int_0^\infty \frac{\mathrm{d}p\,\mathrm{d}k\,\mathrm{d}p'}{(2\pi T)^3}\psi_m(p)$$
$$\times\ \left(\delta(p'-p-k')\gamma(p';p,k)F_n(p';p,k')\frac{X(x)}{T} - \frac{1}{2}(p\to p')\right), \tag{56}$$

where we define F_n as the value of F evaluated at ψ_n, instead of at χ/T.

To obtain the parametric dependence of the bulk viscosity we define the dimensionless quantities

$$\begin{aligned}\bar{\gamma} &= \frac{\mu^4}{T^6}\gamma \\ \delta(p'-p-k') &= \frac{1}{T}\delta\left(\frac{p'-p-k'}{T}\right)\end{aligned} \tag{57}$$

in terms of which, and extracting the factor $X(x)/T$, the right-hand side of equation (56) turns into

$$\frac{X(x)}{T}\frac{T^5}{\mu^4}\langle\psi_m|\frac{\delta\bar{C}}{\delta f}|\psi_n\rangle, \tag{58}$$

where the energy dimension is explicit, since C_{mn} is now a function of the ratios of momenta over temperature alone. One can then check the dimension of the projected Boltzmann equation

$$\int \frac{\mathrm{d}^3 p}{(2\pi T)^3} S(p) = -\frac{T^6}{\mu^4}\frac{\delta\bar{C}(\chi/T)}{\delta f} \tag{59}$$

that matches the defining equation (40) above.

Finally, the parametric dependence of the viscosity, following from equation (42), is given as

$$\zeta = \frac{m_s^4}{T}\langle\bar{S}|\left(\frac{\delta\bar{C}}{\delta f}\right)^{-1}|\bar{S}\rangle. \tag{60}$$

5.3. Numerical evaluation

Once the parametric dependence of the bulk viscosity is known, all that remains is to evaluate a numerical factor. A subtle point comes in choosing an appropriate trial function family ψ_i, so that all integrals converge appropriately in both their infrared (IR) and ultraviolet (UV) domains. For example, the natural family of orthonormal functions in the interval $(0,\infty)$, the Laguerre functions, would yield

$$\psi^m(p) = \frac{\sqrt{2}\pi \mathrm{e}^{-p/2}}{p} L^m(p) \tag{61}$$

with conventional L^m Laguerre polynomials. The $1/p^2$ from two such functions, together with a $1/p$ from the Bose–Einstein factor $(\mathrm{e}^p - 1)^{-1} \to p^{-1}$ and our splitting function $\gamma \propto p^2$, make the integral in equation (56) infrared divergent. Fortunately, it is not necessary to use an orthonormal function family[3], as long as one is not interested in the function χ itself, but only on its projection to obtain the transport coefficient. This can be shown with minimum linear algebra by changing basis from an orthonormal to an arbitrary ψ_i family function. We therefore choose

$$\psi_m(p/T) = \frac{(p/T)^m}{(1+(p/T)^{m+5})}. \tag{62}$$

[3] We thank Guy Moore for this observation.

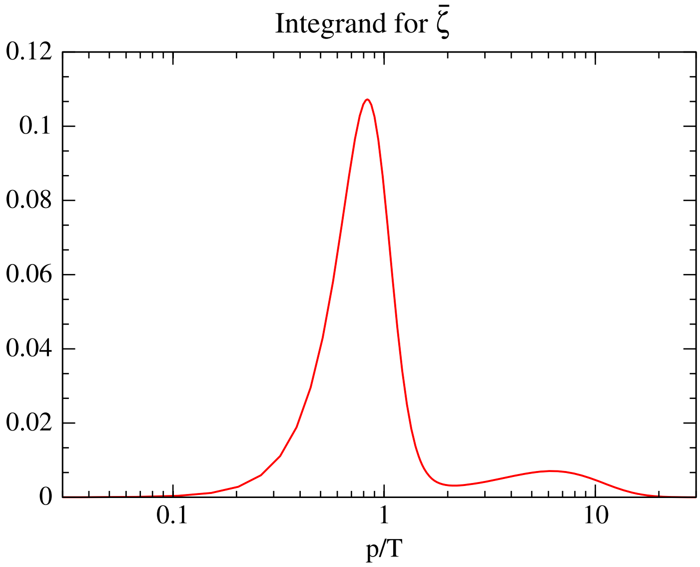

Figure 2. Integrand for the last integral in equation (56) yielding the dimensionless $\bar{\zeta}$, showing IR and UV integrability.

Table 1. Convergence of the pre-coefficient of ζ with the number of functions. First column: number of functions in the family. Second column: result with zero-mode subtraction based on the function $E_p p^2$. Third column: result with zero-mode subtraction based on the function $E_p p^2 f_p(1+f_p)$. This pre-coefficient can be thought of as $\bar{\zeta} = \zeta(m_s = 1, T = 1) = (T/m_s^4)\zeta$.

m	$\bar{\zeta}_1$	$\bar{\zeta}_2$
1	0.009 78	0.009 3
3	0.010 9	0.010 57
5	0.011 0	0.010 67
7	0.011 0	0.010 70

One should also notice that there is a zero mode of the collision integral, visible in the last line of equation (55), that vanishes when $\psi = p$ for $p = p' + k'$. The collision operator is not invertible, but as observed in [22], this does not suppose a problem as one can add to C an arbitrary constant λ times the projector over this zero mode

$$C \to C + \lambda |E_p p^2\rangle\langle E_p p^2|$$

since its projection over the source in equation (60) vanishes. Note that a factor of $f^0(1+f^0)$ can be multiplied to the vector $|E_p p^2\rangle$ without numerically affecting the result, as in [22].

In table 1 we show the fast convergence with the number of functions employed (size of the linear system). The integration is performed with a Gaussian grid; there is no sensitivity to UV or IR cutoffs, as shown in figure 2, nor to the parameter λ that fixes the zero-mode subtraction. In the table, however, we give two sets of numbers, showing that the result is essentially equivalent should the factor $f_0(1+f_0)$ be omitted in the zero-mode subtraction.

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6. Discussion

We have established that the bulk viscosity coefficient associated to the normal fluid component of a cold CFL superfluid is given by

$$\zeta_{\text{CFL}} = 0.011\frac{m_s^4}{T} \tag{63}$$

at first order in the conformal breaking parameter m_s^2/μ^2. This is a remarkable result for several reasons. First, it is much smaller than the shear viscosity already reported in [2]. The reason for being so is that the two coefficients are governed by different sorts of processes (collinear splitting for bulk viscosity, large angle collisions for shear viscosity), which occur at different rates. Second, at the order we computed it is seen to be independent of the chemical potential and the superconducting gap Δ. Third, due to the dynamics being dominated by the superfluid phonon, which is a Goldstone boson which remains always massless, it is not exponentially suppressed as might be thought of based on a calculation involving gapped degrees of freedom, such as, for example, those due to quarks [8] or to kaons [11]. Thus, it is the leading contribution at very small temperatures $T \ll m, \Delta$, where m is the energy gap associated to the lightest massive mode of the CFL phase. This is the temperature regime that we call 'cold'.

The result in equation (63) could have been anticipated from the mean free path for small angle collisions discussed in our previous work [2], namely

$$\lambda_{\text{small}} \propto \frac{\mu^4}{T^5}. \tag{64}$$

A quick estimate of the parametric dependence of the bulk viscosity would be

$$\zeta \propto \lambda_{\text{small}} \times n \times \langle p \rangle \times \mathcal{C}^2 \propto \frac{m_s^4}{T} \tag{65}$$

in terms of the phonon number density $n \propto T^3$ and average momentum $\langle p \rangle \propto T$, and of the conformal breaking parameter $\mathcal{C} = \left(\frac{1}{3} - c_s^2\right) \sim m_s^2/\mu^2$. This immediately yields equation (63) up to the numerical factor. Note that the shear viscosity, however, has a different parametric behavior with the temperature, as small angle collisions are very inefficient for transferring transverse momentum. This is seen in the calculation in [2] by the appearance of near-zero modes that appear in the computation of the $2 \to 2$ collision operator and make collinear splitting irrelevant there[4].

While our bulk viscosity result diverges in the limit $T \to 0$, it should be kept in mind [2] that when the temperature diminishes the mean free path of the phonon becomes large, and at some point the hydrodynamical description of the phonon fluid is meaningless (one rather has free streaming of phonons). Then only the perfect superfluid with no dissipation remains. For astrophysical applications, and assuming that the radius of the compact star is of the order of $R \sim 10$ km, this happens at $T \sim 0.06$ MeV [2].

Although we have computed the bulk viscosity in a fluid at rest, it is possible to extract from our results the frequency-dependent bulk viscosity needed for astrophysical

[4] The *exact* zero mode appearing in equation (55), however, is not a separation from equilibrium as it maintains the detailed balance relation $C[f_0 + \text{zero}] = 0$ and can be subtracted. This explains the failure of relations one could have guessed such as $\zeta \propto m_s^4\eta/\mu^4$, that seem to fail analogously in ϕ^4 theory [37] but hold in the quark–gluon plasma [22].

applications. Based on our results, a quick estimate of a so defined frequency-dependent bulk viscosity has recently appeared [38]. The results are very interesting and suggest that the rate of equilibration of bulk distortions in a hypothetical quark star, at physically relevant frequencies, receive contributions not only from weak equilibration processes but also from the phonon splitting processes that we have studied here [38]. Although quark–gluon equilibration times are short, the phonon system is described by the weakly coupled effective Lagrangian of Son, and the bulk viscosity is proportional to the (small) scale violating parameter. This corrects one's first intuition about neglecting strong interaction phenomenology altogether in the belief that strong interactions should permanently be in equilibrium.

There is also another subtle point. In the existing literature where one needs a bulk viscosity coefficient to perform the analysis of the fate of the r-modes, the computation is performed by analyzing the energy dissipated after a compression or rarefaction over one period, $\tau = 2\pi/\omega$, where ω is the frequency of the fluctuation. Thus

$$\left\langle \frac{\mathrm{d}E_{\mathrm{dis}}}{\mathrm{d}t} \right\rangle = \frac{\zeta}{\tau} \int_0^\tau \mathrm{d}t \, (\nabla \cdot \mathbf{V})^2 \, , \tag{66}$$

where $\mathbf{V}$ is the hydrodynamical velocity. The resulting bulk viscosity coefficient is then given also as a function of ω. While this relation is valid for a normal fluid, it should be generalized for a relativistic superfluid. In a superfluid there are at least three bulk viscosities, and unless the remaining coefficients vanish for CFL quark matter, they should contribute to dissipation in an oscillatory compression or rarefaction of the system, and should contribute to the right-hand side of equation (66).

It is thus urgent that a computation of the remaining viscosities of the CFL superfluid should take place, as well as a careful study of its low temperature hydrodynamics. Both are required for the study of the r-modes of a hypothetical compact star made of CFL quark matter. Let us point out that the relevance of the existence of several viscosities in a superfluid neutron star has only been emphasized in very recent publications [39]–[41]. In particular, only in [41] have all the bulk viscosity coefficients in a neutron superfluid been computed.

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