

THE COMPUTATIONAL PROBLEM OF USING OWA OPERATORS

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Abstract

In this paper we will analyze some computational problems related with the use of OWA operators as information aggregation. In particular we will concentrate on ordered hierarchical aggregations of OWA operators as defined in [6].

Key words: Aggregation operators, combinatorial optimization.

1 Introduction and preliminaries

Ordered Weighted Averaging (OWA) operators were firstly proposed by Yager in [13] and studied subsequently by many other authors both from a theoretical and a practical point of view (see for instance [4,6,8,14]). It is well known that T-norms and T-conorms represent aggregation operators that generalize the notion of conjunction and disjunction of classical logic with, in particular, the min operator being the maximal T-norm and the max operator being the minimal T-conorm (see [10]). OWA operators verify the nice property of filling the gap between min and max. Intuitively, then, by means of OWA operators we can go from conjunction (intersection) to disjunction (union) in a continuous way.

T-norms and T-conorms are also associative. Thus, given any T-norm or T-conorm $F(x, y)$ and given any n values (a_1, \dots, a_n) , we can apply the same operative definition of $F(x, y)$ to obtain $F(a_1, \dots, a_n)$. More in details, we see that we can evaluate such value $F(a_1, \dots, a_n)$ either as $F(F(a_1, \dots, a_{n-1}), a_n)$ or $F(a_1, F(a_2, \dots, a_n))$. Consequently, since we can evaluate in time $O(1)$, i.e. in constant time, the value $F(a, b)$ for any pair a, b we can evaluate $F(a_1, \dots, a_n)$ in time $O(n)$, i.e. in linear time.

1.1 OWA operators

To simplify the formalization of OWA operators we make use of the notion of sorting permutation of a list.

If $L = \{a_1, a_2, \dots, a_n\}$ is a list of numbers, a sorting permutation σ for L is any permutation of the elements of L that produces a list $\sigma(L) = \{a_{\sigma(1)}, \dots, a_{\sigma(n)}\}$ verifying

$$a_{\sigma(i)} \geq a_{\sigma(j)}$$

for all $i \leq j$.

DEFINITION 1.1 An OWA operator of dimension n is an aggregation operator ϕ that has an associated list of weights $W = \{w_1, \dots, w_n\}$ such that

1. $w_i \in [0, 1]$ for all $1 \leq i \leq n$
2. $\sum_{i=1}^n w_i = 1$

3. for any $L = \{a_1, a_2, \dots, a_n\}$ and its corresponding $\sigma(L) = \{a_{\sigma(1)}, \dots, a_{\sigma(n)}\}$

$$\phi(L) = \sum_{i=1}^n w_i a_{\sigma(i)}.$$

□

In view of the above definition, it can be immediately verified that OWA operators are commutative, monotone and idempotent. Moreover, the value $\phi(a_1, \dots, a_n)$ needs an $O(n \log n)$ time to be evaluated since the n input numbers must be sorted.

As previously commented, for any OWA operator ϕ we have

$$\min(a_1, \dots, a_n) \leq \phi(a_1, \dots, a_n) \leq \max(a_1, \dots, a_n) \quad (1.1)$$

Two significative measures are associated with OWA operators of dimension n .

(m1) The first measure is called *orness* and it estimates how close an OWA operator is to the max operator. In details the degree of orness is defined as

$$\text{orness}(\phi) = \frac{1}{n-1} \sum_{i=1}^{n-1} (n-i)w_i$$

As shown in [13],

$$\text{orness}(\phi) \leq \text{orness}(\phi')$$

when their associated weights $\{w_1, \dots, w_n\}$ and $\{w'_1, \dots, w'_n\}$ verify that

$$\sum_{j=1}^i w_j \leq \sum_{j=1}^i w'_j \quad \forall i = 1, \dots, n \quad (1.2)$$

The converse of the above is obviously not true. This property gives in fact an intuitive idea of when an OWA operator must be more an "or" than one other OWA operator.

Dual to the measure of *orness* is the measure of *andness* defined as

$$\text{andness}(\phi) = 1 - \text{orness}(\phi)$$

which therefore estimates how close an OWA operator is to the min operator.

(m2) The second measure called *dispersion* estimates the degree to which all aggregations are used equally, i.e. the degree to which an OWA operator is close to the simple average operator. The degree of dispersion is defined as

$$\text{disp}(\phi) = - \sum_{i=1}^n w_i \ln w_i$$

2 Ordered hierarchies of OWA operators

2.1 Hierarchical aggregations

In some recent scientific investigations the notion of hierarchical aggregation of information has been introduced along with some characteristic theorems (see [3,4]). Intuitively, hierarchical aggregations are aggregations of chunks of information which in turn represent aggregated information. The practical consequences of such hierarchical aggregations are quite interesting. If we have aggregation maps of very big dimensions, hierarchical aggregations will allow us to deal with sub-aggregation operators of smaller dimensions, whose computational jobs can be parallelized. Thus, it will be possible to obtain a significant speed up of the whole aggregation process. Moreover, hierarchical aggregations allow us to treat input data in different ways according to the underlying semantic of the aggregation process. Input data can be seen as divided into clusters, whose equivalence is decided according to an ad hoc aggregation operator, and in turn these resulting representatives can be aggregated by yet another ad hoc aggregation operator.

2.2 Hierarchical aggregations of OWA operators

If the input data are real numbers then by introducing the natural ordering on the reals we can talk about "ordered" hierarchical aggregation rules. This type of hierarchical aggregation arises very naturally in use of Yager's OWA operators as aggregation operators.

Formally, let $\phi_0, \phi_1, \dots, \phi_c$ be $c+1$ OWA operators such that

- ϕ_0 has dimension c ;
- ϕ_i has dimension h_i for any $i = 1, 2, \dots, c$;
- $\sum_{i=1}^c h_i = n$

Let w_0, \dots, w_c be the weights associated to ϕ_0 , and for all $i = 1, \dots, c$ let $w_{i,1}, \dots, w_{i,h_i}$ be the weights associated to ϕ_i .

DEFINITION 2.1 The ordered hierarchical composition of $\phi_0, \phi_1, \dots, \phi_c$ is defined by

$$\phi_0(\phi_1, \dots, \phi_c)(a_1, \dots, a_n) = \phi_0(\phi_1(a(1), \dots, a(h_1)), \dots, \phi_c(a(n-h_c+1), \dots, a(n)))$$

for all n -uples (a_1, \dots, a_n) .

DEFINITION 2.2 If a property P that holds for $\phi_0, \phi_1, \phi_2, \dots, \phi_c$ holds for the ordered hierarchical aggregation $\phi_0(\phi_1, \dots, \phi_c)$ as well, we will say that P propagates under ordered hierarchical aggregation.

As proven in [4] we have that the property of being an OWA operator propagates under ordered hierarchical aggregation as well as Monotonicity, Idempotency, Commutativity.

3 On the degrees of Orness and Dispersion

Given a fixed ordered hierarchical aggregation $\phi = \phi_0(\phi_1, \dots, \phi_c)$ based upon $c+1$ OWA operators, Yager's orness measure takes the expression

$$\text{orness}(\phi) = \phi\left(\frac{n-1}{n-1}, \frac{n-2}{n-1}, \dots, \frac{1}{n-1}, 0\right) = \phi_0(z_1, \dots, z_c, \dots, z_n)$$

where

$$z_i = \phi_i\left(\frac{n - (\sum_{j=1}^{i-1} h_j + 1)}{n-1}, \dots, \frac{n - \sum_{j=1}^i h_j}{n-1}\right) = \sum_{m=1}^{h_i} \frac{n - (\sum_{j=1}^{i-1} h_j + m)}{n-1} w_{i,m} = \frac{n - \sum_{j=1}^{i-1} h_j}{n-1} + \frac{\sum_{m=1}^{h_i} (h_i - m) w_{i,m}}{n-1} = \frac{n - \sum_{j=1}^{i-1} h_j}{n-1} + \left(\frac{h_i - 1}{h_i}\right) \frac{\text{orness}(\phi_i)}{n-1}$$

for all $i = 1, \dots, c$, with h_i the dimension of each ϕ_i , and $(w_{i,1}, \dots, w_{i,h_i})$ its associated weights. Hence,

$$\text{orness}(\phi) = \sum_{i=1}^c \frac{(h_i - 1) \text{orness}(\phi_i) + n - \sum_{j=1}^i h_j}{n-1} w_{0,i}$$

Obviously, changes in ϕ_i 's weights for $j = 1, \dots, c$ in such a way that $\text{orness}(\phi_j)$ does not decrease for any $j = 1, \dots, c$ will never make $\text{orness}(\phi)$ decrease. However, increasing $\text{orness}(\phi_0)$ does not necessarily increase $\text{orness}(\phi)$ as shown by the following example.

Example 3.1 Let ϕ_0, ϕ_1 be two OWA operators of dimension c such that their associated weights are, respectively, $w_{0,j} = 1, w_{0,j} = 0$ for all $j \neq 2$ and $w_{1,j} = w_{1,j} = 1/2, w_{1,j} = 0$ for all $j \neq 1, 3$. Then, $\text{orness}(\phi_0) = \text{orness}(\phi_1)$. Let now ϕ be the minimum rule (i.e., $\text{orness}(\phi) = 0$ for all $i = 1, \dots, c$). Then the difference between $\text{orness}(\phi_0(\phi_1, \dots, \phi_c))$ and $\text{orness}(\phi_1(\phi_0, \dots, \phi_c))$ will still depend on the relative sizes h_1 and h_3 of OWA operators ϕ_1, ϕ_3 .

The dispersion of the hierarchical aggregation can be computed as follows.

Denote by ϕ an OWA operator obtained from an ordered hierarchical aggregation of OWA operators $\phi_0, \phi_1, \dots, \phi_c$ with dimensions c, h_1, \dots, h_c respectively. Clearly,

$$\begin{aligned} \text{Disp}(\phi) &= - \sum_{j=1}^c \sum_{i=1}^{h_j} w_{0,j} w_{j,i} \log(w_{0,j} w_{j,i}) \\ &= - \sum_{j=1}^c \sum_{i=1}^{h_j} (w_{0,j} w_{j,i} \log w_{0,j} + w_{0,j} w_{j,i} \log w_{j,i}) \\ &= - \sum_{j=1}^c w_{0,j} \log w_{0,j} - \sum_{j=1}^c w_{0,j} \text{Disp}(\phi_j) \\ &= \text{Disp}(\phi_0) + \sum_{j=1}^c w_{0,j} \text{Disp}(\phi_j) \end{aligned}$$

Therefore, the dispersion of the hierarchical aggregations depends directly upon the dispersions of the given OWA operators.

In both cases it is therefore clear that once the operators ϕ_0 and ϕ_1, \dots, ϕ_c are fixed then the orness and dispersion values of the hierarchical aggregation depend upon the particular ordering of the c OWA operators ϕ_1, \dots, ϕ_c .

It is then natural to ask the following questions:

- (1) How can one quickly find an ordering of those c OWA aggregated operators which maximises (resp. minimises) the dispersion value of the hierarchical aggregation?
- (2) How can one quickly find an ordering of those c OWA aggregated operators which maximises (resp. minimises) the orness value of the hierarchical aggregation?

The above mentioned computational problems have already been proposed and characterized in [5,7].

4 Maximizing and minimizing dispersion and orness

Let ϕ_0 be an OWA operator of dimension c and let $\Phi = \{\phi_i : 1 \leq i \leq c\}$ be a given set of OWA operators and let h_1, \dots, h_c be their respective dimensions.

4.1 Dispersion

We start by providing polynomial algorithms to maximize (resp. minimize) the dispersion of the hierarchical aggregation.

Let π be a permutation of the indices $\{1, 2, \dots, c\}$ such that

$$i \leq j \text{ if and only if } w_{0,\pi(i)} \geq w_{0,\pi(j)}$$

Intuitively, for $i = 1, 2, \dots, c$, $w_{0,\pi(i)}$ is the i -th weight in non-increasing order.

Let ϕ_1, \dots, ϕ_c be a given ordering of the OWA operators in Φ .

The following lemmas hold.

LEMMA 4.1 The value $\text{Disp}(\phi) = \text{Disp}(\phi_0) + \sum_{j=1}^n w_{0j} \text{Disp}(\phi_j)$ is maximum if and only if the ordering ϕ_1, \dots, ϕ_n verifies

$$i \leq j \text{ if and only if } \text{Disp}(\phi_{\pi(i)}) \geq \text{Disp}(\phi_{\pi(j)}).$$

LEMMA 4.2 The value $\text{Disp}(\phi) = \text{Disp}(\phi_0) + \sum_{j=1}^n w_{0j} \text{Disp}(\phi_j)$ is minimum if and only if the ordering ϕ_1, \dots, ϕ_n verifies

$$i \leq j \text{ if and only if } \text{Disp}(\phi_{\pi(i)}) \leq \text{Disp}(\phi_{\pi(j)}).$$

In view of the above Lemmas and from the well-known result (see, e.g., [1]) that any given n numbers can be sorted in $O(n \log n)$ time, we can claim that the following theorem is true.

THEOREM 4.1. Given an OWA operator ϕ_0 of dimension c and a set of c OWA operators Φ and given their dispersions, there exists a $O(c \log c)$ algorithm which produces an ordering of Φ maximizing (or minimizing) the dispersion of the hierarchical aggregation. \square

4.2 Orness

The problem of maximizing or minimizing the orness of the hierarchical aggregation at a first sight appears to be quite more difficult from a computational point of view and its (polynomial) solution definitely more subtle. However such a problem can be reduced to the classical combinatorial assignment problem for which there exists a $O(c^2)$ algorithm (see [7] for more details).

Briefly, we have that

- denoted with $\phi_1, \phi_2, \dots, \phi_n$ the OWA operators in Φ , and
- denoted for simplicity with δ_{ij} the contribution to the orness of ϕ of the operator ϕ_i once it is chosen as the j -th in the hierarchical aggregation, then
- the problem of maximizing (resp. minimizing) the orness of the hierarchical aggregation is equivalent to the problem of finding a permutation π of the rows of the matrix

$$M = \begin{pmatrix} \delta_{11} & \delta_{12} & \dots & \delta_{1c} \\ \delta_{21} & \delta_{22} & \dots & \delta_{2c} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{n1} & \delta_{n2} & \dots & \delta_{nc} \end{pmatrix}$$

such that the sum of the elements of the main diagonal of

$$M_\pi = \begin{pmatrix} \delta_{\pi(1)1} & \delta_{\pi(1)2} & \dots & \delta_{\pi(1)c} \\ \delta_{\pi(2)1} & \delta_{\pi(2)2} & \dots & \delta_{\pi(2)c} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{\pi(c)1} & \delta_{\pi(c)2} & \dots & \delta_{\pi(c)c} \end{pmatrix}$$

is maximum (resp. minimum), i.e. such that

$$\sum_{i=1}^c \delta_{\pi(i)i}$$

is maximum (resp. minimum).

The above is the classical assignment problem (also known as the bipartite weighted matching problem (see [12], chapter 11) and can be solved by the so-called Hungarian method (see [11]) in time $O(c^2)$. More recently in [9] it has been proven that by using special data structures it is possible to improve the above bound to $O(c^2 \log c)$.

As proven in [9] we have that $O(c \log c)$ algorithms can be given for the most significative cases in practice. Since each one of the given c OWA operators represents partial information to be aggregated into only one index, it is common to define them in such a way that either

- they contain the same amount of information. In this case, all OWA operators in Φ will have the same dimension h such that $n = h \cdot c$; or
- they treat their inputs with the same degree of optimism (pessimism). In this case, all OWA operators will have the same degree of orness.

5 Final Comments

In this paper, we continue our research work aimed at a full characterization of hierarchical aggregation operators (see [2,3]) by focusing our attention on the hierarchical aggregations of OWA operators (see [4]) and to the related computational problems ([6]). Since the property of being an OWA operator propagates under ordered hierarchical aggregations and since the orness and dispersion of the obtained OWA operator depend upon the given ordering of the aggregated OWA operators, we have studied how this values can be maximized or minimized. In particular, we have provided polynomial algorithms which solve the general problem for both the dispersion and the orness.

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IN GROUP DECISION MAKING UNDER LINGUISTIC PREFERENCES AND FUZZY LINGUISTIC QUANTIFIERS

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ABSTRACT

In this paper some results on group decision making under linguistic preferences and fuzzy linguistic quantifiers are presented. Assuming a set of individual linguistic preferences, representing the preferences of the particular individuals, we develop a solution method for the decision process. We define a linguistic ordered weighted averaging operator, and use it for deriving a collective linguistic preference where the weights are defined using a fuzzy linguistic quantifier. Finally, we use the concept of nondominated alternatives for obtaining a set of maximal and nondominated alternatives from the collective linguistic preference, that is, the solution to the decision process.

Keywords: Group decision making, linguistic preferences, linguistic quantifiers.

Introduction

Since human judgments including preferences are often vague, fuzzy logic plays an important role in decision making. Several authors have provided interesting results on group decision making or social choice theory with the help of fuzzy sets. They have proved that fuzzy sets provided a more flexible framework for discussion group decision making [15, 8, 9, 11].

In a fuzzy environment it is supposed there exists a finite set of alternatives $X = \{x_1, \dots, x_n\}$ as well as a finite set of individuals $N = \{1, \dots, m\}$, and each individual $k \in N$ provides his preference relation on X , i.e., $P^k \subset X \times X$, and $\mu_{P^k}(x_i, x_j)$ denotes the degree of preference of alternative x_i over x_j , $\mu_{P^k}(x_i, x_j) \in [0, 1]$.

Sometimes, however, an individual could have a vague information about the preference degree of the alternative x_i over x_j , and can not estimate his preference with an exact numerical value. Then a more realistic approach may be to use linguistic assessments instead of numerical values, that is, to suppose that the variables (preference relations) which participate in the problem are assessed by means of linguistic terms [5, 17, 2, 4, 6, 12]. A scale of certainty expressions (linguistically assessed) would be presented to the individual which could then use to describe his degree of certainty in a preference.

Assuming a set of individual linguistic preferences, we present a solution method for the decision process. We define a collective linguistic ordered weighted averaging (L-OWA) operator, and use it for deriving a collective linguistic preference where the weights are defined using a fuzzy linguistic quantifier. Finally, we use the concept of nondominated alternatives for obtaining a set of maximal nondominated alternatives from the collective linguistic preference, that is, the solution to the decision process.

This model seems to be more human consistent with the social choice in an imprecise environment. The linguistic decision process can be summarized in the figure 1.

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PREFACE

Soft computing constitutes a collection of disciplines which include fuzzy logic, neural networks, genetic algorithms and probabilistic reasoning. It is fast emerging as a tool to help computer-based intelligent systems mimic the ability of the human mind to employ modes of reasoning that are approximate rather than exact. The basic thesis of soft computing is that precision and certainty carry a cost and that intelligent systems should exploit, wherever possible, the tolerance for imprecision and uncertainty. Considerable success has been achieved in the application of this principle, especially with the use of fuzzy logic, in the development of a large number of intelligent control systems. These types of systems have appeared in applications as diverse as large scale subway controllers and as small as video cameras. We are now at a juncture where the ideas implicit in soft computing will begin to have significant impact in many other domains of application, this is especially true of information-related applications such as database and information retrieval. Another area where we shall see great use being made of fuzzy logic technology is in the construction of "things that think", for example a coat that can adapt its thermal properties to different weather conditions.

This volume focuses on the current state of soft computing, especially the fuzzy logic component. It comprises seven sections. The first section consists of three articles on fuzzy logic and genetic algorithms. The second section focuses on the issue of learning in soft computing. The third section, on hybrid and fuzzy systems, concerns itself with the use of fuzzy logic in a number of different paradigms employed in soft computing. The fourth section is devoted to decision and aggregation techniques. The fifth section concentrates on the use of fuzzy technologies in database systems. The sixth section has ten articles on foundational issues in fuzzy set theory. The final section describes a number of applications in which fuzzy logic plays a major role.

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