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MODELLING AND FORECASTING A

BALANCE OF PAYMENTS



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ABSTRACT

This paper reports on the development of methods for modelling and forecasting a complete national Balance of Payments (BP), employing an extended version of the Box and Jenkins approach to time series analysis. The full BP is treated, findings on the properties typical of such data are described, new approaches to modelling capital flow variables, net flow (balance) variables and the official foreign exchange reserves flow are offered. Illustrations are taken from the authors' study for the Spanish case with monthly data.

RESUMEN

Este trabajo informa sobre el desarrollo de métodos para modelizar y preveer una Balanza de Pagos (BP) nacional, empleando una versión extendida del enfoque de análisis de series temporales Box y Jenkins. Se trata una BP completa y esto permite describir las propiedades típicas de tales datos. En este artículo se presentan nuevos enfoques para modelizar las variables flujo de capital, las variables de flujo neto o de saldo y las reservas de divisas. Las ilustraciones ofrecidas en este artículo han sido extraídas del estudio realizado por los autores para el caso español con datos mensuales.

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1 Introduction

Economic data usually come in the form of structured systems. Accounting systems constitute one of the most important kinds of structured systems of statistical data for the description of economies. One very basic kind of accounting system, used by economies as small as one person and as large as a set of national economies, is the Balance of Payments (BP). It includes all of the monetary flows between the economy in question and the corresponding external economy.

A BP has the following structure:

$$\nabla \text{ Reserves} \equiv \text{Surplus on Current Account} + \text{Surplus on Capital Account}$$

$$\text{Surplus on Current Account} \equiv \Sigma \text{ Current Account Receipts} - \Sigma \text{ Current Account Payments}$$

$$\text{Surplus on Capital Account} \equiv \Sigma \text{ Capital Inflows} - \Sigma \text{ Capital Outflows}$$

where $\nabla \equiv 1 - B$ is the difference operator, B the backshift operator, Reserves is a stock measured at the end of time periods, all other variables are flows, and the Σ operator in each case is taken over the categories of transactions designed into the data collection system and/or selected as partial aggregates for the purposes of a given case.

Thus the BP will have: (1) a sampling interval, e.g. monthly, annual, (2) definite categories of data under each Σ and (3) data coverage of all variables present in the above identities. Usually one would also want to cover further "surplus" variables at lower levels of aggregation, e.g., the trade balance (\equiv Receipts from goods exports - Payments for goods imports).

In the research underlying this paper, *Río (1995)*, a systematic modeling of a complete national BP was undertaken, for the first time as far as the authors are aware. The Spanish Balance of Payments (SBP) was treated. The balance of payments has been studied in many papers, though in none of them does this classic accounting system seem to have been given a complete treatment. See, for example, *Padilla (1987)*, *Sebastián (1991)*, *Yang (1992)* or *Wilson (1994)*. The complete

coverage given to the SBP permits a classification of all of the statistical series in terms of common properties. Three types of time series which show common group characteristics are suggested: (1) current account variables, (2) capital account variables and (3) net flow and balance variables.

The underlying time series analysis approach taken in this research is that pioneered by Box and Jenkins (1970, 3rd ed 1994) as extended in the professional literature and in our practice. All models are estimated using the Exact Maximum Likelihood algorithm of *Mauricio (1995)* and seasonal series are treated with the generalized seasonal methodology of *Gallego and Treadway (1996)*.

A system of variables such as in a BP should clearly be treated with a multivariate stochastic model, even when, as in the case of the monthly SBP, the model is diagonal. Coupling may be found through further research, but is not found to be outstanding in the present case. The operational objective of this research is to construct a model, or set of models, with which to perform monthly forecast and monitor services for the SBP and on which further research on relationships with non-BP data can improve.

This objective is coherent with the overall objectives of the "Forecast and Monitor Service for the Spanish Economy" (SPS:EE for Servicio de Previsión y Seguimiento de la Economía Española) as described in *Treadway (1994)*.

Current account variables seem to follow processes typical of economic flows, already largely described in *Treadway (1984)*; the logarithms of these variables are $I(2)$ at zero frequency, are frequently $I_1(1)$ at some or all seasonal frequencies f , have some low-order MA structure, little or no AR structure and anomalies arise from step incidents in levels of variables (e.g. effects of exchange rate changes). However, both capital account and net flow (balance) variables have special characteristics: (1) variable signs, (2) evidence of heteroskedasticity not associated with level, suggesting probable (3) non-linearity and non-normality, and (4) numerous extreme values. These statistical properties that are present in capital account and balance variables, including current account balance variables, have made such series difficult to model. A similar conclusion can be found in *Lin (1992)*. The present article reports on methodological issues that have arisen in the

treatment of such atypical time series, some examples of which are presented from *Río (1995)*. The ways these issues have been faced appear to be useful in modelling the SPB and may well turn out to be useful in a wider variety of similar applications.

The treatment suggested for capital flows arises from the, we believe plausible, idea that the corresponding capital stock follows a loglinear ARIMA process. The stock, valued at the historical value of its accumulated flows, is observed conditional on the presample initial value, to be efficiently estimated jointly with the ARIMA and intervention parameters, when the latter are appropriate. ARIMA model specification is conducted conditional on an initial guess of the initial stock, which is a Box-Cox (1964) origin parameter, but such specification is found to be insensitive to the initial value when this is taken to make for low enough rates of the flow/stock ratio in sample. The logarithm of such stocks appears to follow an IMA(2,1) process in most cases, breaking down to a random walk with possibly nonzero mean stepsize when the sample is short.

The treatment suggested for balance variables is to model the strictly positive components on each side of the BP as stochastic, the flows themselves for current account variables and the stocks for the capital account variables, and to model the balance variable by application of the accounting identity. This approach is also suggested for the reserves flow.

This paper is structured as follows. In Section 2 we present the suggested methods for dealing with capital flow variables. In Section 3, a strategy for modelling net flow balance variables is presented. In Section 4, the analysis of a single variable as an aggregate of components, applied to the study of the Reserves variable, is presented. A section summarizing the conclusions and a graphics appendix close the paper.

2 Capital Flow Variables

The time series in the capital account are flow variables. This is also the case for time series from the current account, but the former behave in a substantially different way from the latter. Typical current account series present, after taking their (natural) logarithms, approximately good

statistical properties: linearity, normality and homoskedasticity. However, typical capital account flow series take positive values at some times and negative values at other times and show heteroskedasticity not associated with level. It is impossible to take the logarithms of these series; the Box-Cox transformation can not be used directly. As an example of the typical properties that capital account flow series show, see the plots for $FLLC_t$ and $\nabla FLLC_t$, corresponding to Long-Term Credits Received by Spain, in the graphics appendix.

2.1 A Methodology For Analyzing a Supposedly Lognormal Stock by Using a Sample of Flows

Start with the assumption that the logarithm of the stock series S_t , corresponding to the flow series F_τ ($\tau = -\infty, \dots, t$), and defined as:

$$S_t \equiv \sum_{\tau=-\infty}^t F_\tau \equiv S_0 + \sum_{\tau=1}^t F_\tau ,$$

follows a linear, normal, homoskedastic process of the ARIMA type. The flow series F_t ($t = 1, \dots, N$) is observed and a new time series is defined:

$$Z_t \equiv \sum_{\tau=1}^t F_\tau ,$$

so that:

$$S_t = Z_t + m ,$$

where $m = S_0$ is the unknown value for the initial stock, which is taken as a parameter to be estimated.

Box and Cox (1964) suggest a family of transformations defined in terms of two real parameters (λ, m) :

$$Z_t^{(\lambda, m)} = \frac{(Z_t + m)^\lambda - 1}{\lambda} , \text{ with } Z_t > -m ,$$

where $\lambda = 0$, $m = 0$ occurs in many economic variables and corresponds to the logarithm of the

variable. When analyzing economic time series, this transformation family is known to be useful for achieving approximately linear, normal, and homoskedastic processes.

Estimation of λ and m through maximum likelihood, when $Z_t^{(\lambda, m)}$ follows a univariate stochastic ARIMA process, is easy in theory and even more so in the present case, since λ does not have to be estimated; it is fixed at 0.0 in order to deal with the logarithm of the stock series.

For given values of (λ, m) , $Z_t^{(\lambda, m)}$ is here assumed to follow an ARIMA(p, d, q) process. The possibility of seasonal structure is ignored in the present treatment, since the capital account series do not exhibit seasonality. A generalization to cope with this possibility is straightforward.

Box and Jenkins (1970, Appendix A7.4) specify the relevant likelihood function for estimation purposes as follows:

$$L[\phi, \theta, \sigma_a^2 | Z^{(\lambda, m)}] = (2\pi\sigma_a^2)^{-n/2} |M_n^{(p, q)}|^{-1/2} \exp \left\{ \frac{-S(\phi, \theta)}{2\sigma_a^2} \right\} , \quad (2.1)$$

where:

$$S(\phi, \theta) = \sum_{t=-\infty}^n [a_t | Z^{(\lambda, m)}, \phi, \theta]^2 , \quad (2.1.1)$$

$Z^{(\lambda, m)}$ is the $N \times 1$ vector of (transformed) observations, $n = N - d$ is the number of effective observations after taking differences, $\sigma_a^2 M_n^{(p, q)}$ is the $n \times n$ covariance matrix of the stationary (transformed and differenced) series and $[a_t | Z^{(\lambda, m)}, \phi, \theta] = [a_t]$ is the mathematical expectation of a_t conditioned on the observed values and on the parameters of the ARMA structure. In practice, the values $[a_t]$ can be calculated through backcasting as described by Box and Jenkins (1970, pp. 215-216). For a stationary model, the $[a_t]$ values are negligible in magnitude beyond $t < 1 - Q$ for some integer Q , so that the summation in (2.1.1) starts at $t = 1 - Q$ instead of at $t = -\infty$.

The corresponding expression of the log-likelihood function is:

$$\ln L[\phi, \theta, \sigma_a^2 | \mathbf{Z}^{(\lambda, m)}] = -\frac{n}{2} \ln(2\pi\sigma_a^2) - \frac{1}{2} \ln |\mathbf{M}_n^{(p, q)}| - \frac{S(\phi, \theta)}{2\sigma_a^2} \quad (2.2)$$

In order to obtain maximum likelihood estimates of the Box-Cox (λ, m) parameters, the likelihood function in (2.1) is extended through its multiplication by the jacobian of the transformation of the original data into the transformed data. Hence, the resulting log-likelihood function is obtained by adding to (2.2) the logarithm of the jacobian, which can be shown to be:

$$(\lambda - 1) \sum_{t=d+1}^N \ln(Z_t + m) .$$

The extended log-likelihood function is then:

$$\begin{aligned} \ln L[\phi, \theta, \sigma_a^2, \lambda, m | \mathbf{Z}] &= -\frac{n}{2} \ln(2\pi\sigma_a^2) - \frac{1}{2} \ln |\mathbf{M}_n^{(p, q)}| - \frac{S(\phi, \theta, \lambda, m)}{2\sigma_a^2} + \\ &+ (\lambda - 1) \sum_{t=d+1}^N \ln(Z_t + m) , \end{aligned} \quad (2.3)$$

where:

$$S(\phi, \theta, \lambda, m) = \sum_{t=1-Q}^n [a_t | \mathbf{Z}, \phi, \theta, \lambda, m]^2 \quad (2.3.1)$$

The notation has been expanded in order to recognize the new dependencies, and the lower limit $1-Q$ of the summation in (2.3.1) has been incorporated. Note that the matrix $\mathbf{M}_n^{(p, q)}$ depends on ϕ, θ , though this has not been made explicit in the notation, but does not depend on λ, m .

Given values for λ and m , $S(\phi, \theta, \lambda, m)$ in (2.3.1) can be minimized numerically following Box and Jenkins (1970) to obtain least squares estimates for ϕ, θ and σ_a^2 ; alternatively, minus (2.2) can be minimized numerically in order to obtain exact maximum likelihood estimates for ϕ, θ and σ_a^2 . The former option, implemented in TASTE, a program suite designed, built and used by the SPS:EE research team, was employed in this study, because, at the time of the work underlying this paper,

no implementation of the latter option was available in our computing environment (a new version of TASTE is expected to be available soon with facilities for MVE as treated by *Mauricio (1995)*; this will undoubtedly lead to improvements). Using either option, one obtains estimates $\hat{\phi}(\lambda, m), \hat{\theta}(\lambda, m), \hat{\sigma}_a^2(\lambda, m)$, where:

$$\hat{\sigma}_a^2(\lambda, m) = \frac{S(\hat{\phi}(\lambda, m), \hat{\theta}(\lambda, m), \lambda, m)}{n} .$$

When these estimates are substituted into the log-likelihood function (2.3):

$$\ln L(\lambda, m | \mathbf{Z}) = -\frac{n}{2} \ln[2\pi\hat{\sigma}_a^2(\lambda, m)] - \frac{1}{2} \ln |\mathbf{M}_n^{(p, q)}| - \frac{n}{2} + (\lambda - 1) \sum_{t=d+1}^N \ln(Z_t + m) \quad (2.4)$$

If one further fixes λ at 0.0, which will be the case if one wants to deal with the logarithm of the original series, the terms in (2.4) that depend on m can be gathered:

$$F(m | \mathbf{Z}) = -n \ln \hat{\sigma}_a(m) - \sum_{t=d+1}^N \ln(Z_t + m) \quad (2.5)$$

Hence, in order to estimate m , the following two-step procedure may be used: (i) given a trial value for m , build a US model for S_t and obtain $\hat{\sigma}_a(m)$, (ii) maximize (2.5) over m , and (iii) iterate until m converges. Once an estimate for m , say \hat{m} is found, a $100(1 - \alpha)\%$ confidence interval may be constructed by using the fact that:

$$\ln L(\hat{m}) - \ln L(m) \leq \frac{1}{2} \chi_1^2(\alpha) ,$$

where $\chi_1^2(\alpha)$ represents the value of a χ^2 variate with one degree of freedom that leaves a probability equal to α to the left, $\ln L(\hat{m})$ is the value of the log-likelihood function at \hat{m} and $\ln L(m)$ is the value of the log-likelihood function at m .

In summary, the following procedure is suggested in order to build a US model for a stock

series S_t by using its corresponding flow series F_t :

(1) It is assumed that $\lambda = 0.0$ and an initial value for m , say m^0 , is taken.

(2) A stock series is built: $S_t^0 = Z_t + m^0$. A univariate analysis of $\ln S_t^0$ is performed and the parameters of the detected ARIMA structure are estimated using either least squares or exact maximum likelihood. $\hat{\sigma}_a(m^0)$ is obtained.

(3) The function (2.5) is maximized over m using a Quasi-Newton method. Steps (1)-(3) are repeated successively until the sequence of estimates for m converges.

(4) Using the final value for m , the stock series is built and a diagnostic-checking procedure is carried out, reformulating the model and going back to estimate m again if necessary.

2.2 Example: Long-Term Credits Received by Spain (FLLC)

This series is the balance obtained as the difference between foreign currency receipts derived from loans received and their corresponding amortizations. Such loans are provided by non-residents to both the resident public and private sectors for a term of more than one year. All kinds of credits are included, that is, commercial, financial and other credits; see *Notas al Boletín Estadístico del Banco de España (1989)*.

The data for the illustration have been taken from the Bank of Spain's Registro de Caja statistics for the time period 4/86-12/92. This sample has been chosen, because it is longer than the samples available for the new SBP statistics. The sample begins in 4/86, because, in the previous month, this class of transactions was liberalized. The liberalization was not completed at this date; there were other changes in the rules that regulated this kind of credits later; see *Eguidazu (1991)* and *Alejano (1994)*.

The first two pages of the graphics appendix report on this series. Plots of the flow series are shown ($FLLC_t$ and $\nabla FLLC_t$ in Figures A.1 and A.2) as well as plots of the stock series based on the estimated initial stock. Plots of $\ln SLLC_t$ (Figure A.3) with its acf, of $\nabla \ln SLLC_t$ (Figure A.4) and $\nabla^2 \ln SLLC_t$ (Figure A.5) with their acf and pacf, and of the residuals for the final model (Figure A.6),

with their acf and pacf, are also shown.

The $FLLC_t$ series shows an upward trend and is clearly heteroskedastic. On applying a regular difference, a homoskedastic series does not appear to arise, i.e. $\nabla FLLC_t$ is centred but is clearly heteroskedastic. Also, the flow $FLLC_t$ takes positive values in some months and negative values in others.

The estimated $SLLC_t$ series (graph not shown), is heteroskedastic. By applying a logarithmic transformation, the estimated series $\ln SLLC_t$ is obtained: it seems to be homoskedastic and shows a rising trend since 5/87. Hence, a regular difference is applied.

The estimated $\nabla \ln SLLC_t$ series does not appear to be mean-stationary; this is confirmed by the plot of its acf.

The estimated $\nabla^2 \ln SLLC_t$ series appears clearly to be stationary in mean. It is interesting to compare this plot with that of the $\nabla FLLC_t$ series; the two series are centred, but $\nabla^2 \ln SLLC_t$ seems to be homoskedastic while $\nabla FLLC_t$ is heteroskedastic. Observe that $\nabla \ln SLLC_t \approx \frac{FLLC_t}{SLLC_{t-1}}$; flow heteroskedasticity is treated essentially by scaling with the lagged stock. An MA(1) model with a positive parameter is tentatively specified for $\nabla^2 \ln SLLC_t$.

The resulting IMA(2,1) model is estimated and the estimation results show an estimated MA parameter significantly different from zero and clearly invertible. The following atypical values are observed in the residual plot: 6/87 (+2.5 $\hat{\sigma}_a$), 7/87 (+2.1 $\hat{\sigma}_a$), 8/87 (+2.0 $\hat{\sigma}_a$), 7/88 (-2.4 $\hat{\sigma}_a$), 2/89 (-2.0 $\hat{\sigma}_a$), 5/91 (+2.3 $\hat{\sigma}_a$) and 8/92 (-2.6 $\hat{\sigma}_a$). The extreme values in 6-8/87, 2/89 and 5/91 coincide with dates in which a change took place in legal regulations. Therefore, certain intervention terms are included at these dates. Other intervention analyses carried out for other dates with extreme values (impulse in 11/86, steps in 7/88 and 8/92) do not reveal influential incidents.

The estimated model seems to be adequate: the residual plot (Figure A.6) seems to be compatible with the hypothesis of white noise. The diagnostic tools, acf and pacf, do not suggest a model reformulation. The coefficient at the 4th lag with negative sign stands out; this is due to a distortion generated by noninfluential extreme values of opposite sign separated by four months, in

3 Net Flow (Balance) Variables

In the study of a BP and in general in the study of any accounting system, it is useful to study the net flow variables for the different categories of operations. The balance variables in a BP measure the net flows of payments for goods, services, rents, investments, for example.

The SBP balance series do not manifest properties typical of the current account flow series, but resemble the capital flow variables. Some balance variables present a trend, but not all. Some current account balances present clear seasonality and the majority of them are heteroskedastic, but the heteroskedasticity is not of the kind that one can treat with a Box-Cox transformation directly. Extreme values are also important in these series.

It seems reasonable in this case to use the models for the components on each side of the balance and treat the balance variable by applying its identity. This approach has two important characteristics that favor it: (1) to forecast the balance variable not only is the historical information about the variable itself used, but the historical information about each of the payment and receipt (liability and asset) flows is also used and (2) the heteroskedasticity, non-linearity and non-normality of the balance series are treated by the application of the identity defined on its loglinear component variables.

Assuming that a balance variable D , e.g., the deficit on current account, is made up of n_p payment variables P_i ($i = 1, \dots, n_p$) and n_y receipt variables Y_j ($j = 1, \dots, n_y$), then the corresponding identity is:

$$D_t \equiv \sum_{i=1}^{n_p} P_{it} - \sum_{j=1}^{n_y} Y_{jt}$$

Lowercase characters will be used for logarithms, so $p_{it} \equiv \ln P_{it}$ and $y_{jt} \equiv \ln Y_{jt}$. Univariate or multivariate models for the components are assumed to be available and these allow one to calculate

point forecasts from origin N at lead time $\ell > 0$ for p_i , $\hat{p}_{iN}(\ell)$, and for y_j , $\hat{y}_{jN}(\ell)$. The corresponding forecast errors are denoted by $e_N^p(\ell) \equiv p_{iN+\ell} - \hat{p}_{iN}(\ell)$ and $e_N^y(\ell) \equiv y_{jN+\ell} - \hat{y}_{jN}(\ell)$. These errors should have an approximate Normal distribution with zero mean and variance-covariances that can be calculated from the underlying models.

The point forecast for the balance variable at origin N and lead time ℓ , is calculated by application of the identity:

$$\hat{D}_N(\ell) \equiv \sum_{i=1}^{n_p} \hat{P}_{iN}(\ell) - \sum_{j=1}^{n_y} \hat{Y}_{jN}(\ell)$$

The forecast error $D_{N+\ell} - \hat{D}_N(\ell)$ thus is $e_N^D(\ell)$. The theoretical variance of this forecast error is:

$$\begin{aligned} V[e_N^D(\ell)] &\equiv \sum_{i=1}^{n_p} V[e_N^p(\ell)] + \sum_{k=1}^{n_p} \sum_{i=1, i \neq k}^{n_p} Cov[e_N^p(\ell), e_N^p(\ell)] + \\ &+ \sum_{j=1}^{n_y} V[e_N^y(\ell)] + \sum_{k=1}^{n_y} \sum_{j=1, j \neq k}^{n_y} Cov[e_N^y(\ell), e_N^y(\ell)] - \\ &- 2 \sum_{i=1}^{n_p} \sum_{j=1}^{n_y} Cov[e_N^p(\ell), e_N^y(\ell)] \end{aligned}$$

In order to evaluate the variances of the forecast errors on the levels of the component series, a first-order Taylor approximation is used:

$$\begin{aligned} e_N^p(\ell) &\approx \hat{p}_{iN}(\ell) e_N^p(\ell) & \forall i = 1, \dots, n_p \\ e_N^y(\ell) &\approx \hat{y}_{jN}(\ell) e_N^y(\ell) & \forall j = 1, \dots, n_y \end{aligned} \quad (3.1)$$

In practice, only a few contemporary correlations are detected between components of the SBP. The corresponding contemporary covariance values are used in calculating the variance of $e_N^D(\ell)$. The estimated variance of these forecast errors diminishes when these non-zero correlations are taken into account.

Note that, though the variances of the forecast errors of the component series do not depend on the forecast origin, the variance of the forecast error of the balance variable depends in general on N and not only on the lead time ℓ . For this reason, in monitoring operations, in which the forecast error at lead time $\ell = 1$ is of interest, it is convenient to compare the distribution of the standardized errors

$$\frac{e_{t-1}^D(1)}{\sqrt{V(e_{t-1}^D(1))}} \quad (3.2)$$

with the $N(0,1)$ distribution. The sample standardized error series of the balance variables, calculated as in (3.2), should present characteristics consistent with the hypothesis of white noise.

The Spanish Deficit on Trade Account ($DTA = \text{payments minus receipts}$) is used to illustrate the ideas of this section; see the graphics appendix. The plots corresponding to the original series (Figure A.7) are shown for the sample 1/86-12/92 with the acf, as are the first regular difference of the original series (Figure A.8) and the plot of standardized residuals (Figure A.9) with the corresponding acf and pacf. The DTA series shows an upward trend, is heteroskedastic and shows a seasonal pattern. The ∇DTA series is centred but it is clearly heteroskedastic. The characteristics of heteroskedasticity, non-linearity and non-normality are taken into account by modelling the variable in terms of its loglinear components. The residual plot has characteristics that appear compatible with the hypothesis of white noise.

4 Official Reserves

The BP for any economic agent contains a net flow variable that should be modelled by application of the BP identity itself, taking all other variables in the identity to be modelled as jointly stochastic. In a national BP for a contemporary economy with a central bank that sets the national exchange rate by accepting the resulting official foreign exchange reserves changes, the change in such reserves plays this role. If the central bank were to allow free-market determination of the

national exchange rate with no change in its holding of foreign exchange, then another BP variable would play this role. In the SBP, the first case is the one characterizing the data.

The Official Reserves change series is a typical capital account flow variable, with variable sign and heteroskedasticity not associated with level.

Both of the strategies discussed in Sections 2 and 3 are combined here in the treatment of this variable: (1) the models for the SBP components and the accounting identity are used, hence modeling the Reserves flow variable as a linear combination of the rest of the SBP variables and (2) for the components from the capital account, the methodology developed in Section 2 is used.

The BP identity may be written:

$$\nabla R_t \equiv SBCC_t + FNK_t \quad (4.1)$$

where $SBCC_t$ is the surplus on current account (receipts minus payments) and FNK_t is the surplus on capital account without Official Reserves (liabilities flows minus assets flows).

The series $SBCC_t$ is made up of n_y receipts variables Y_j ($j = 1, \dots, n_y$) and n_p payments variables P_i ($i = 1, \dots, n_p$):

$$SBCC_t \equiv \sum_{j=1}^{n_y} Y_{jt} - \sum_{i=1}^{n_p} P_{it} \quad (4.1.1)$$

The series FNK_t is made up of n_e liability flow variables FP_k ($k = 1, \dots, n_e$) and n_a asset flow variables FA_z ($z = 1, \dots, n_a$). The surplus on capital account satisfies the identity:

$$FNK_t \equiv \sum_{k=1}^{n_e} FP_{kt} - \sum_{z=1}^{n_a} FA_{zt} \quad (4.1.2)$$

Substituting (4.1.1) and (4.1.2) into (4.1):

$$\nabla R_t \equiv \left[\sum_{j=1}^{n_y} Y_{jt} - \sum_{i=1}^{n_p} P_{it} \right] + \left[\sum_{k=1}^{n_e} FP_{kt} - \sum_{z=1}^{n_a} FA_{zt} \right]$$

For each of the capital account flow variables, the corresponding stock series is estimated as described in Section 2 and a univariate model for the logarithm of the estimated stock variable is constructed. For the logarithms of the current account variables, univariate models are built as described in Section 3. These models are used to calculate the point forecast from origin N for lead time $\ell > 0$ for each component of the SBP as well as its associated forecast error variance.

The point forecast for R_t from origin N for lead time $\ell > 0$ is calculated by using the identity:

$$\hat{R}_N(\ell) = \hat{R}_N(\ell-1) + \left[\sum_{j=1}^{n_y} \hat{Y}_{jN}(\ell) - \sum_{i=1}^{n_p} \hat{P}_{iN}(\ell) \right] + \left[\sum_{k=1}^{n_e} \nabla \hat{SP}_{kN}(\ell) - \sum_{z=1}^{n_a} \nabla \hat{SA}_{zN}(\ell) \right] .$$

where the capital account flow variables are treated in terms of the corresponding estimated stock variables. In this case, the operator ∇ affects ℓ and not N . The associated forecast error is:

$$\begin{aligned} e_N^R(\ell) &= R_{N+\ell} - \hat{R}_N(\ell) \\ &= e_N^R(\ell-1) + \left[\sum_{j=1}^{n_y} e_N^{Y_j}(\ell) - \sum_{i=1}^{n_p} e_N^{P_i}(\ell) \right] + \left[\sum_{k=1}^{n_e} \nabla e_N^{SP_k}(\ell) - \sum_{z=1}^{n_a} \nabla e_N^{SA_z}(\ell) \right] . \end{aligned}$$

where $e_N^{Y_j}(\ell)$, $e_N^{P_i}(\ell)$, $e_N^{SP_k}(\ell)$, $e_N^{SA_z}(\ell)$, $e_N^R(\ell-1)$ are forecast errors corresponding to the receipt (Y_j) and payment (P_i) variables from the current account, to the estimated liability (SP_k) and asset (SA_z) stock variables from the capital account, and to the stock variable corresponding to the Official Reserves flow, respectively. The variance of $e_N^R(\ell)$ is calculated:

$$\begin{aligned} V[e_N^R(\ell)] &= V[e_N^R(\ell-1)] + V\left[\sum_{j=1}^{n_y} e_N^{Y_j}(\ell) - \sum_{i=1}^{n_p} e_N^{P_i}(\ell)\right] + V\left[\sum_{k=1}^{n_e} \nabla e_N^{SP_k}(\ell) - \sum_{z=1}^{n_a} \nabla e_N^{SA_z}(\ell)\right] \\ &\quad - 2 \sum_{k=1}^{n_e} \text{Cov}[e_N^R(\ell-1), e_N^{SP_k}(\ell-1)] + 2 \sum_{z=1}^{n_a} \text{Cov}[e_N^R(\ell-1), e_N^{SA_z}(\ell-1)] \\ &\quad + 2 \sum_{j=1}^{n_y} \sum_{k=1}^{n_e} \text{Cov}[e_N^{Y_j}(\ell), \nabla e_N^{SP_k}(\ell)] - 2 \sum_{j=1}^{n_y} \sum_{z=1}^{n_a} \text{Cov}[e_N^{Y_j}(\ell), \nabla e_N^{SA_z}(\ell)] \\ &\quad - 2 \sum_{i=1}^{n_p} \sum_{k=1}^{n_e} \text{Cov}[e_N^{P_i}(\ell), \nabla e_N^{SP_k}(\ell)] + 2 \sum_{i=1}^{n_p} \sum_{z=1}^{n_a} \text{Cov}[e_N^{P_i}(\ell), \nabla e_N^{SA_z}(\ell)] . \end{aligned}$$

To evaluate the variances of the forecast errors defined on level series, a first order Taylor approximation is used as in Equation (3.1).

The third and last case presented in the graphics appendix corresponds to the graphics report on this variable (Figures A.10, A.11 and A.12). The sample period shown in this graphic is relatively short, from 4/91 to 6/93. The length of this sample is conditioned in practice by the short sample period used in the analyses of certain capital account series subject to recent liberalization. In Figure A.10 the plot of the original series ∇R_t is shown. Figure A.11 presents the series $\nabla^2 R_t$ with its acf and pacf. The residual plot, with the corresponding correlograms, is shown in Figure A.12. In this plot the standardized forecast errors at lead time $\ell = 1$ are drawn, which take the form:

$$\frac{e_{t-1}^R(1)}{\sqrt{V(e_{t-1}^R(1))}} \quad (4.2)$$

The SBP components were investigated in search for relationships, but only a little contemporary correlation was detected between these series. The values of the contemporary cross correlations detected in the SBP series have been considered in the calculation of $V[e_{t-1}^R(1)]$ in Equation (4.2). The characteristic of heteroskedasticity shown in the plots of ∇R_t and $\nabla^2 R_t$ (Figures A.10 and A.11) does not appear in the residual plot (Figure A.12). This plot presents characteristics

that are in agreement with the white noise hypothesis: the plot is centred, the mean does not differ significantly from zero and there are not many extreme values. In both the acf and pacf there is no evidence of unmodelled structure.

5 Conclusions

The methods and analytical strategies described in this paper have been in use for several years for generating reports on the SBP published monthly in *Previsión y Seguimiento de la Economía Española*; see Treadway (1994). When the full SBP forecast and monitor system was first set up in early 1993, the forecast for Official Reserves implied that these would go to zero by mid-year; this is equivalent to forecasting a major devaluation of the peseta, which in fact arose in May 1993. A univariate model of Official Reserves did not give such results. This strongly suggests that the methodological contributions offered in this paper are relevant in practice.

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GRAPHICS APPENDIX

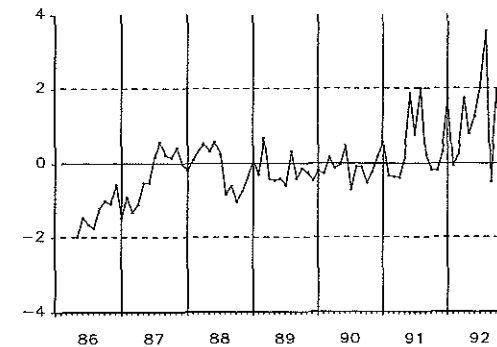


Figure A.1 Standardized FLLC_t series ($\bar{w} = 41701.06$ (9008.30), $\delta_w = 81074.67$)

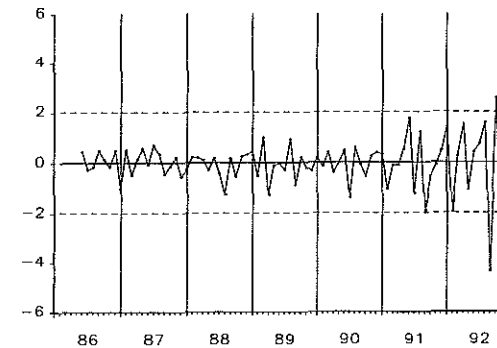


Figure A.2 Standardized VFLLC_t series ($\bar{w} = 5505.09$ (8543.48), $\delta_w = 76415.20$)

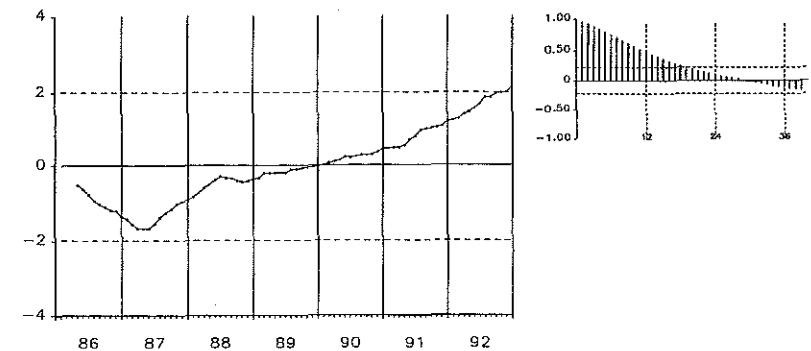


Figure A.3 Standardized lnSLLC_t series ($\bar{w} = 14.04$ (0.04), $\delta_w = 40.24\%$) and acf

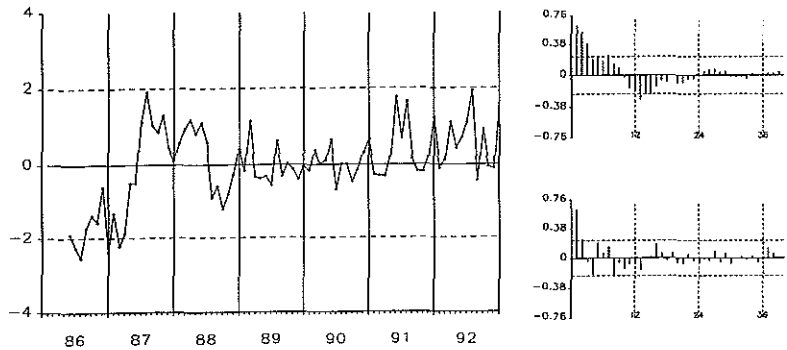


Figure A.4 Standardized $VlnSLLC_t$ series ($\bar{w} = 1.36\%$ (0.35%), $\hat{\sigma}_w = 3.10\%$) and acf and pacf

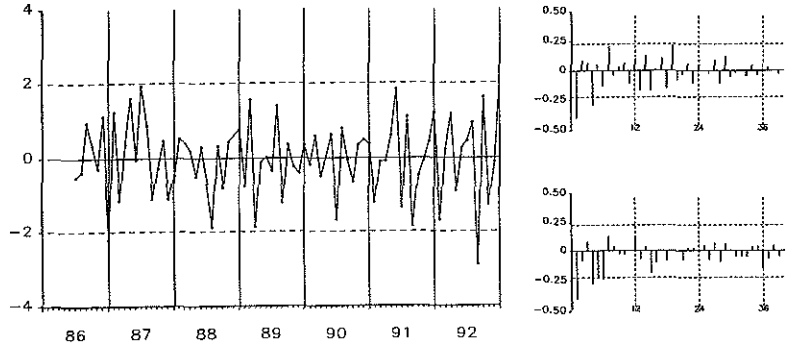


Figure A.5 Standardized $V^2lnSLLC_t$ series ($\bar{w} = 0.14\%$ (0.29%), $\hat{\sigma}_w = 2.56\%$) and acf and pacf

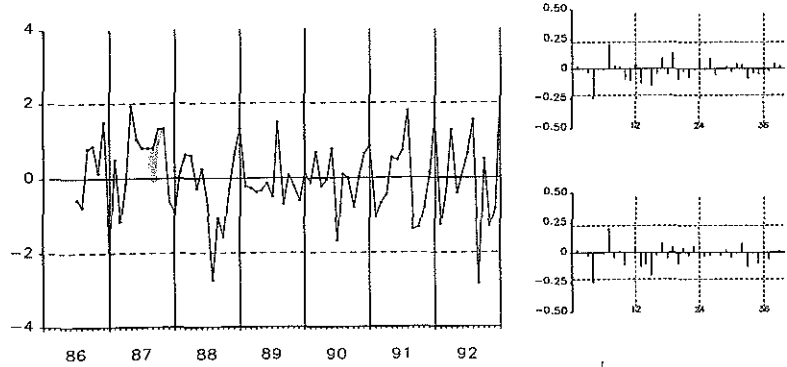


Figure A.6 Standardized residuals of the SLLC Model ($\bar{w} = 0.24\%$ (0.23%), $\hat{\sigma}_w = 2.04\%$) and acf ($Q(39) = 23.3$) and pacf

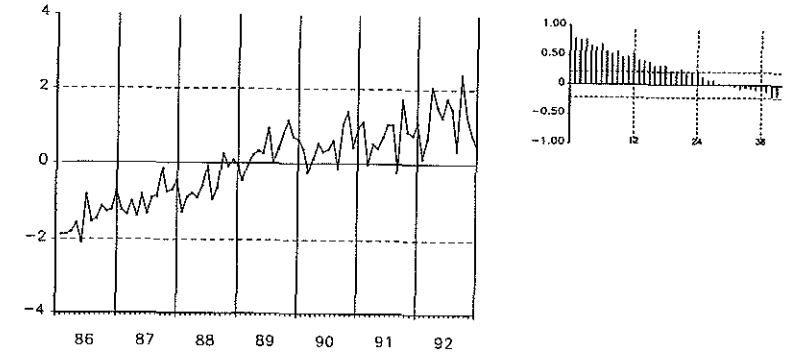


Figure A.7 Standardized DTA_t series ($\bar{w} = 202.43$ (10.56), $\hat{\sigma}_w = 96.80$) and acf

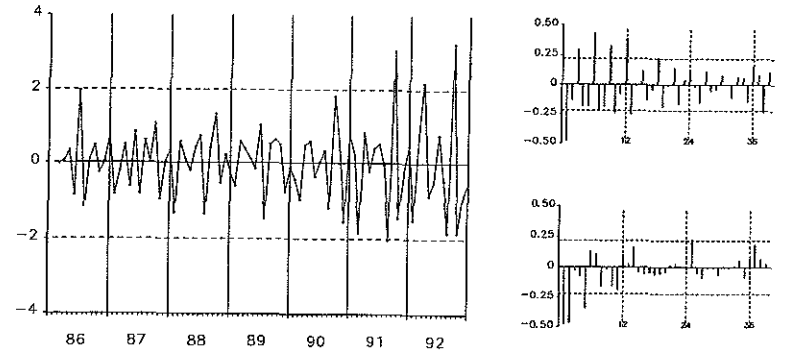


Figure A.8 Standardized $VDTA_t$ series ($\bar{w} = 2.69$ (6.66), $\hat{\sigma}_w = 60.69$) and acf and pacf

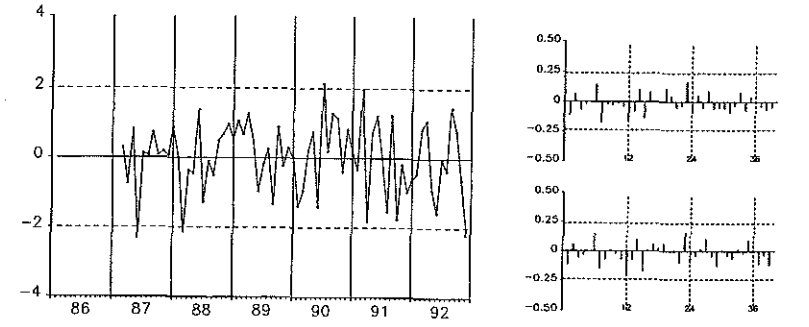


Figure A.9 Standardized residuals of the DTA model ($\bar{w} = -0.02$ (0.11)) and acf ($Q(39) = 24.0$) and pacf

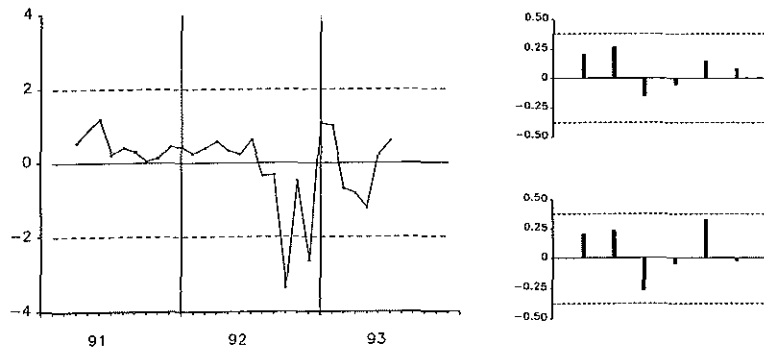


Figure A.10 Standardized VR_t series ($\bar{w} = -38338.93$ (70202.92), $\hat{\sigma}_w = 371478.96$) and acf and pacf.

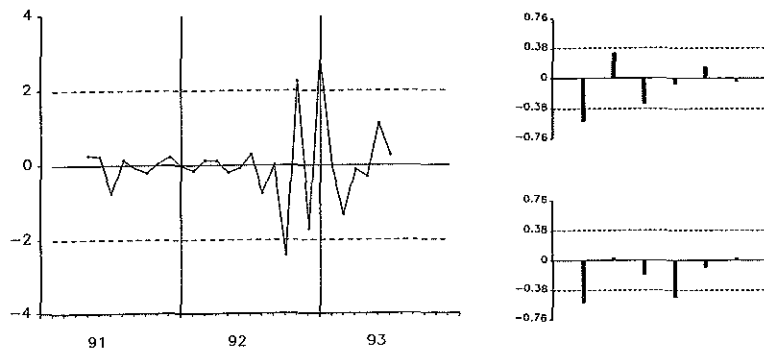


Figure A.11 Standardized V^2R_t series and acf and pacf.

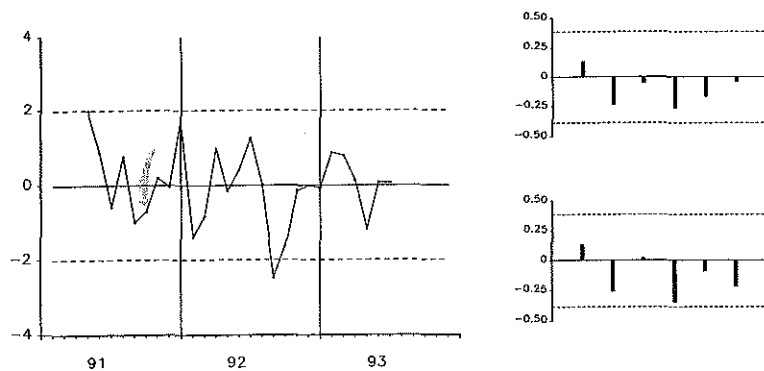


Figure A.12 Standardized residuals of the R model ($\bar{w} = 0.07$ (0.15)) and its acf ($Q(6) = 5.7$) and its pacf.

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