

Nonlinear model of c -number confined Dirac quarks

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We investigate the effect of spin-dependent forces in a recently proposed approach to the problem of extended particles with structure. The forces are introduced by means of two different fourth-order interactions of Fermi type, a vector coupling and a combination of a scalar and pseudoscalar terms. The mass parameter is 286 (393) MeV in the vector (scalar-pseudoscalar) case and the mass of the spin-zero meson turns out to be 582 (552) MeV, close to that of the η , those of the baryon and the spin-one meson being 1200 and 800 MeV in both cases. As the model has only one flavor, it does not allow other particles.

I. INTRODUCTION

The purpose of this work is the development of a recently proposed nonconventional approach to the problem of extended particles with structure. In a previous paper¹ (henceforth to be referred to as I) the authors presented a mechanism of confinement which combines two basic elements, the use of classical extended particlelike solutions and the representation of the strong interactions by means of direct nonlinear couplings. More precisely, the model defines an extended particle as any particlelike solution of the field equations and does not make use of any intermediate boson as carrier of the strong forces, which are given by four-fermion couplings.

The model is, therefore, nonconventional, since most physicists believe today that quantum chromodynamics, which is clearly based on different ideas, correctly describes the basic features of strong interactions and will achieve a complete and satisfactory picture of hadrons in a near future. Nonetheless, there are at least two good reasons to consider alternative approaches, based on nongauge interactions. First of all, four-fermion couplings could be useful to build phenomenological models in some cases, much as with the Fermi theory of weak interactions. Second, and as the history of science shows, it is wise not to put all the eggs in the same basket. In our case, we could argue, as a third reason, that our approach tries to develop, as an intermediate stage, a classical relativistic mechanics of extended particles by means of particlelike solutions of nonlinear Dirac equations, the nonexistence of which might be one of the reasons for the many difficulties encountered in the quantum theory of extended objects.

The model, whose details are given in I, makes use of six Dirac fields which form two triplets ψ_k and ϕ_k , $k=1,2,3$, the ϕ_k being charge conjugate to the ψ_k , which interact through fourth-order two-body forces. A particle is defined as any finite-order nonsingular solitary-wave or particlelike solution of the field equations. If the ψ_k are interpreted as quarks and the ϕ_k as antiquarks, a striking

parallelism with the experimental situation arises. For instance, there are particles which are bound states of the three ψ_k (or of the three ϕ_k) and of one ψ_k and one ϕ_k , corresponding to the conventional $3q$ ($3\bar{q}$) and $q\bar{q}$ states. But there are no nonzero-triality solutions, as one-field solitary waves or bound states of two ψ_k , two ϕ_k , three ψ_k and one ϕ_k , and so on, which have no analogy in the quark model, as they would correspond to the states q , $2q$, $2\bar{q}$, $3q\bar{q}$, etc.

Curiously enough, the mechanism for which the confinement is a particular case of the triality, is not related to any quantum effect, since the fields are treated as c -numbers. This suggests the possibility that the confinement might be a property of the field equations at the classical level, which perhaps should be investigated before their quantization.

In a more precise way, the main properties of the model are the following.

(1) It makes use of only one kind of quark (only one flavor in the usual parlance).

(2) It gives an explanation of the confinement because the set of solutions of the equations of motion does not contain one-field solitary waves, that is, with only nonvanishing field. In other words, the fields of the quarks cannot manifest themselves as particles, although they appear as constituents of composite systems. The reason for this curious property is that each field acts as a source for the rest of them, much the same as the electric and the magnetic fields in an electromagnetic wave. In fact, as there are neither electric nor magnetic waves we could say that they are confined constituents of an electromagnetic one. It is clear that the existence of static electric and magnetic fields does not spoil this analogy.

(3) It gives an explanation of triality, at least for the low-lying S waves, because in the finite-energy solitary waves the difference between the number of nonvanishing Dirac fields and of their charge-conjugate ones (that is of quarks and antiquarks) is always a multiple of 3.

(4) It depends on only two quantities: a mass parameter m and a coupling constant λ . If $m=390$ MeV and

$\lambda m^2 = 22.98$ the model predicts the following ground-state particles: (i) A three-quark baryon of spin $\frac{3}{2}$ and a mass of 1200 MeV, together with its antiparticle, (ii) a family of quark-antiquark mesons with a spin between 0 and 1 and a mass of 800 MeV, and (iii) two families of two-quark—two-antiquark and three-quark—three-antiquark mesons with the same mass and spin as the previous one.

(5) The model is classical in the sense that, as it has not been quantized, it uses c -number fields and gives a precise space-time description of the particles, although it has not been obtained as any kind of limit $\hbar \rightarrow 0$.

The two important properties (2) and (3) can be expressed as follows: To the statement of the usual quark model that "a hadron is any composite state of quarks, but it happens that there are only zero-triality bound systems of more than one constituent," there corresponds in our model the following one: "A hadron is given by any finite-energy solitary wave of the field equations, but it happens that there are only zero-triality bound solitary waves of more than one Dirac field."

As we see, the main properties which are usually attributed to the quarks are naturally explained. The main drawback of the model is the mass degeneracy and the indeterminacy of the spin in the families of mesons. This is clearly due to the lack of dependence on the spin of the coupling terms in the Lagrangian density. This is important because one of the most conspicuous features of the elementary particles is the strong mass difference between the vector and the pseudoscalar mesons. This suggests the convenience of introducing in the model spin-dependent forces between the quarks. This will be done in the present paper.

In Sec. II we consider the general form of the spin-dependent coupling between the quarks, two special cases being investigated in Secs. III and IV. In Sec. V we make some remarks on the similarity between the strong interactions and the nonlinear effects. Finally in Sec. VI we summarize our results and state our conclusions.

II. SPIN-DEPENDENT FORCES

The Lagrangian density considered in I has three terms: the linear L_1 , the binding L_2 , and the trializing parts,

$$L = L_1 + L_2 + L_3, \quad (1a)$$

$$L_1 = \sum [L_D(\psi_k) + L_{\bar{D}}(\phi_k)], \quad (1b)$$

$$L_2 = \frac{\lambda}{3} [S^2(\psi) + S^2(\phi) + 4S(\psi)S(\phi)], \quad (1c)$$

$$L_3 = \lambda' \sum_{i < j} (\bar{\chi}_{ij} \chi_{ij})^2, \quad (1d)$$

where we recall that ψ_k are three identical classical Dirac fields, ϕ_k are charge conjugate to them, $L_D(\psi)$ and $L_{\bar{D}}(\phi)$ are the usual linear Lagrangian densities with a change of sign in the derivative terms in $\partial_\mu \phi$, and

$$\chi_{ij} = \chi_i - \chi_j, \quad \chi_i = \psi_i + \gamma^5 \phi_i,$$

$$S(\psi) = \sum \bar{\psi}_k \psi_k, \quad S(\phi) = \sum \bar{\phi}_k \phi_k.$$

None of these three terms depends on the spin. In order

to introduce spin dependent forces we must modify the nonlinear terms of the Lagrangian extending its dependence to other bilinear covariant forms:

$$\begin{aligned} P(\psi) &= \sum \bar{\psi}_k \gamma^5 \psi_k, & V^\mu(\psi) &= \sum \bar{\psi}_k \gamma^\mu \psi_k, \\ A^\mu(\psi) &= \sum \bar{\psi}_k \gamma^\mu \gamma^5 \psi_k, & T^{\mu\nu}(\psi) &= \sum \bar{\psi}_k \gamma^\mu \gamma^\nu \psi_k, \end{aligned} \quad (2)$$

and the analogous functions of ϕ_k . Although the class of such Lagrangian densities is very large and depends on as much as 10 constants, in this paper we will only consider two cases, to be called the V and the S - P models, based on the couplings

$$V \text{ model: } V^\mu V_\mu,$$

$$S\text{-}P \text{ model: } S^2 - P^2,$$

the model proposed in I being called from now on the S model. The reason for choosing these cases is the following. Any Lagrangian density based on the couplings S^2 , P^2 , and $V^\mu V_\mu$ gives a theory in which the ratio of the masses of the baryon and the spin-one meson is $\frac{3}{2}$, as it can be shown very easily by using the energy-momentum tensor as is frequently done in this paper. This is no longer true if the terms $A^\mu A_\mu$ and $T^{\mu\nu} T_{\mu\nu}$ are also included. This ratio is close to that of the masses of the $\Delta(\frac{3}{2}^+)$ and the ω or the ρ^0 . Moreover the two cases considered in this work seem to be very adequate since they give a reasonably good value for the mass of the spin-zero meson.

But before proceeding any further, it is convenient to consider the effect of the spin-dependent forces in the case of only one field.^{2,3} It can be shown^{4,5} that the most general form of the Lagrangian density of a Dirac field with a fourth-order self-coupling is

$$L = L_D(\psi) + \lambda [(\bar{\psi}\psi)^2 + z(\bar{\psi}\gamma^5\psi)^2], \quad (3)$$

where we emphasize the necessity of the presence of the term $(\bar{\psi}\psi)^2$ because a theory depending only on the pseudoscalar self-coupling $(\bar{\psi}\gamma^5\psi)^2$ would not lead to the existence of particlelike solutions. For $z=0$ we obtain the Soler model³ and for $z=1$ the Dirac-Weyl equation.⁶ Except in the first case the field equations cannot be factorized in spherical coordinates and to find the localized solitary waves we have to resort to a multipole expansion or to a variational method. It can be shown that the two procedures are equivalent in the calculation of the lowest-wave approximation. If the field ψ is in the S wave it is a combination of the spinors

$$\psi_\uparrow = e^{-i\omega t} \begin{pmatrix} g \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \text{if} \begin{pmatrix} \cos\theta \\ \sin\theta e^{i\phi} \end{pmatrix} \end{pmatrix}, \quad \psi_\downarrow = e^{-i\omega t} \begin{pmatrix} g \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \text{if} \begin{pmatrix} \sin\theta e^{-i\phi} \\ -\cos\theta \end{pmatrix} \end{pmatrix}, \quad (4)$$

where f and g are functions of the radial coordinate r . Using the dimensionless variables

$$\Omega = \omega/m, \quad \rho = mr, \quad (F, G) = \left[\frac{2\lambda}{m} \right]^{1/2} (f, g),$$

we obtain after substitution in the Lagrangian density and integration over the angular variables

$$\int L d^3r = \frac{2\pi}{\lambda m} \int_0^\infty [FG' - GF' - (2/\rho)FG + \Omega(F^2 + G^2) - (G^2 - F^2) + \frac{1}{2}(G^2 - F^2)^2 + z \frac{4}{3} F^2 G^2] \rho^2 d\rho. \quad (5)$$

The variation of F and G leads to the differential radial equations

$$\begin{aligned} G' + (1 + \Omega + F^2 - MG^2)F &= 0, \\ F' + \frac{2}{\rho}F + (1 - \Omega + MF^2 - G^2)G &= 0, \end{aligned} \quad (6)$$

where $M = 1 - 2z/3$. To solve (6) we have followed the same method which was used in many of our quoted references where it is explained in detail. The regular solutions must verify $F(0) = 0$ to avoid the singularity at $\rho = 0$, depending thus only on $G(0)$. It turns out that, in an interval of $G(0)$, the corresponding solutions behave at infinity as $F \rightarrow 0$, $G \rightarrow \pm \sqrt{1 - \Omega}$. The separatrices between the positive and the negative behavior are the only square-integrable solutions and decrease exponentially at infinity. Two different numerical schemes were used, a fourth-order Runge-Kutta and the Hamming predictor-corrector method, with a complete agreement between them. The integrals were calculated by the method of the interpolants of Hermite.

The energy, norm, and spin can be calculated by using the corresponding currents and have the values

$$\begin{aligned} E(z) &= \frac{2\pi}{\lambda m} \mathcal{E}(z), \quad \mathcal{E}(z) = \Omega I_1 + \frac{1}{2} I_2 + \frac{2}{3} z I_3, \\ N(z) &= \frac{2\pi}{\lambda m^2} I_1, \\ S &= \frac{1}{2} N(z), \end{aligned} \quad (7)$$

where

$$\begin{aligned} I_1 &= \int_0^\infty (F^2 + G^2) \rho^2 d\rho, \\ I_2 &= \int_0^\infty (F^2 - G^2)^2 \rho^2 d\rho, \\ I_3 &= \int_0^\infty (F^2 G^2) \rho^2 d\rho. \end{aligned} \quad (8)$$

We will make use in the following of the S -wave form of

$$(i\Gamma^\mu \partial_\mu - m)\Psi + \frac{2}{3}\lambda [(\bar{\Psi}\Delta_\mu\Psi)\Delta^\mu\Psi + (\bar{\Psi}P_{1\mu}\Psi)P_2^\mu\Psi + (\bar{\Psi}P_{2\mu}\Psi)P_1^\mu\Psi] + 2\lambda' \sum_{i < j} (\bar{\Psi}B_{ij}\Psi)B_{ij}\Psi = 0, \quad (12a)$$

which corresponds to the following system of six equations for the fields of the components:

$$\begin{aligned} (i\gamma^\mu \partial_\mu - m)\psi_a + \frac{2}{3}\lambda [V_\mu(\psi) + 2V_\mu(\phi)]\gamma^\mu \psi_a + 2\lambda' \sum (\bar{\chi}_{ak}\chi_{ak})\chi_{ak} &= 0, \\ (-i\gamma^\mu \partial_\mu - m)\phi_a + \frac{2}{3}\lambda [V_\mu(\phi) + 2V_\mu(\psi)]\gamma^\mu \phi_a + 2\lambda' \sum (\bar{\chi}_{ak}\chi_{ak})\gamma^5 \chi_{ak} &= 0. \end{aligned} \quad (12b)$$

the charge-conjugate spinor ϕ which is

$$\begin{aligned} \phi_\dagger &= e^{i\Omega mt} \left[\frac{m}{2\lambda} \right]^{1/2} \begin{bmatrix} G \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ iF \begin{bmatrix} -\sin\theta e^{-i\phi} \\ \cos\theta \end{bmatrix} \end{bmatrix}, \\ \phi_\downarrow &= e^{i\Omega mt} \left[\frac{m}{2\lambda} \right]^{1/2} \begin{bmatrix} G \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ -iF \begin{bmatrix} \cos\theta \\ \sin\theta e^{i\phi} \end{bmatrix} \end{bmatrix}. \end{aligned} \quad (9)$$

As we will show, the radial equations appearing in the models discussed in Secs. III and IV are all of the form (6) with different values of z .

III. VECTOR COUPLING (V MODEL)

Let us consider, in the first place, the modification of L_2 based on the change of the bilinear form S by the vector form V^μ :

$$L_2 = \frac{\lambda}{3} [V_\mu(\psi)V^\mu(\psi) + V_\mu(\phi)V^\mu(\phi) + 4V_\mu(\psi)V^\mu(\phi)]. \quad (10)$$

If we choose to use the field of the hadron $\Psi = (\psi_1, \psi_2, \psi_3, \phi_1, \phi_2, \phi_3)^T$ we may express the Lagrangian as

$$L = L_1 + L_2 + L_3, \quad (11a)$$

$$L_1 = \frac{i}{2} (\bar{\Psi}\Gamma^\mu \partial_\mu \Psi - \partial_\mu \bar{\Psi}\Gamma^\mu \Psi) - m \bar{\Psi}\Psi, \quad (11b)$$

$$L_2 = \frac{\lambda}{3} [(\bar{\Psi}\Delta_\mu \Psi)(\bar{\Psi}\Delta^\mu \Psi) + 2(\bar{\Psi}P_{1\mu} \Psi)(\bar{\Psi}P_2^\mu \Psi)], \quad (11c)$$

$$L_3 = \lambda' \sum_{i < j} (\bar{\Psi}B_{ij}\Psi)^2, \quad (11d)$$

where

$$\Gamma^\mu = \gamma^\mu \otimes A, \quad A = I_3 \otimes \sigma_3, \quad \Delta^\mu = \gamma^\mu \otimes I_6,$$

$$P_1^\mu = \gamma^\mu \otimes P_1, \quad P_2^\mu = \gamma^\mu \otimes P_2,$$

P_1 and P_2 being the projections

$$\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & I_3 \end{bmatrix},$$

and B_{ij} defined as in I. The field equation is

We now follow a process parallel to the one described in I where we refer for details. Let us consider the S -wave solutions of the model, keeping in mind that the algebraic conditions for the existence of the different kind of solutions are the same as in I, the term L_3 being unchanged.

A. Baryonlike solutions

Let us look for solutions of the field equations with the additional condition of vanishing the conjugate fields ϕ_k ,

$$\phi_1 = \phi_2 = \phi_3 = 0,$$

as has been proved in I, the condition for being zero their sources have as consequence the equality of the other three fields

$$\psi_1 = \psi_2 = \psi_3 = \psi, \quad (13)$$

where ψ is now a solution of the equation

$$(i\gamma^\mu \partial_\mu - m)\psi + 2\lambda(\bar{\psi}\gamma_\mu\psi)\gamma^\mu\psi = 0, \quad (14a)$$

which corresponds⁴⁻⁶ to the Dirac-Weyl equation

$$(i\gamma^\mu \partial_\mu - m)\psi + 2\lambda[(\bar{\psi}\psi)\psi + (\bar{\psi}\gamma^5\psi)\gamma^5\psi] = 0 \quad (14b)$$

obtained from the Lagrangian (3) for a value of $z=1$. It is a very interesting equation because it arises in the study of gravitational self-action of a Dirac field.⁷ It is not separable in spherical coordinates but its lowest wave approximation has been studied in detail.⁶ It turns out that there is a family of solutions of the form (4) depending continuously of the frequency with nodeless radial functions F, G solutions of the equations (6) with a coefficient $M = \frac{1}{3}$.

The energy, baryonic norm, and spin take the values

$$\begin{aligned} E &= 3 \left[\frac{2\pi}{\lambda m} \right] \mathcal{E}(1), \quad \mathcal{E}(1) = \Omega I_1 + \frac{1}{2} I_2 + \frac{2}{3} I_3, \\ N &= 3 \left[\frac{2\pi}{\lambda m^2} \right] I_1, \\ \vec{S} &= \left[0, 0, \pm \frac{N}{2} \right], \end{aligned} \quad (15)$$

where I_1, I_2, I_3 are given by (8). The minimum-energy solution was found at $\Omega = 0.899$, with E, N , and the mean square radius having the values

$$\begin{aligned} E &= \frac{6\pi}{\lambda m} 1.4453, \\ N &= \frac{6\pi}{\lambda m^2} 1.0327, \\ \langle r^2 \rangle^{1/2} &= 1.70/m. \end{aligned} \quad (16)$$

There is a slight difference in the fourth significant figure with respect to the quoted reference, the present values being more accurate.

In Fig. 1 we show the curve $\mathcal{E}(\Omega)$ which has a very curious spiral shape. For some values of Ω there are several solutions and for the center of the spirals it seems that there are infinite. In Fig. 2 we can see the radial functions F and G for the lowest minimum of the energy. No solu-

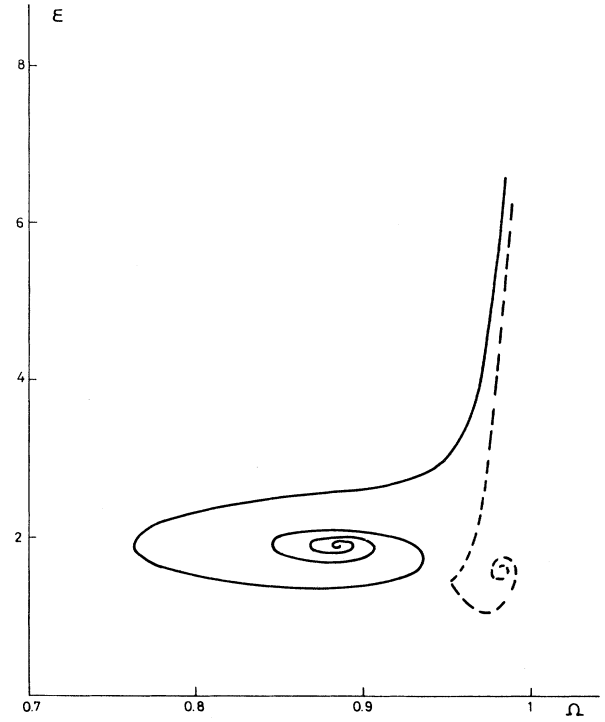


FIG. 1. Shape of the curves $\mathcal{E}(\Omega)$ corresponding to the baryon or the spin-one meson (solid curve) and the spin-zero meson (dashed curve) in the V model.

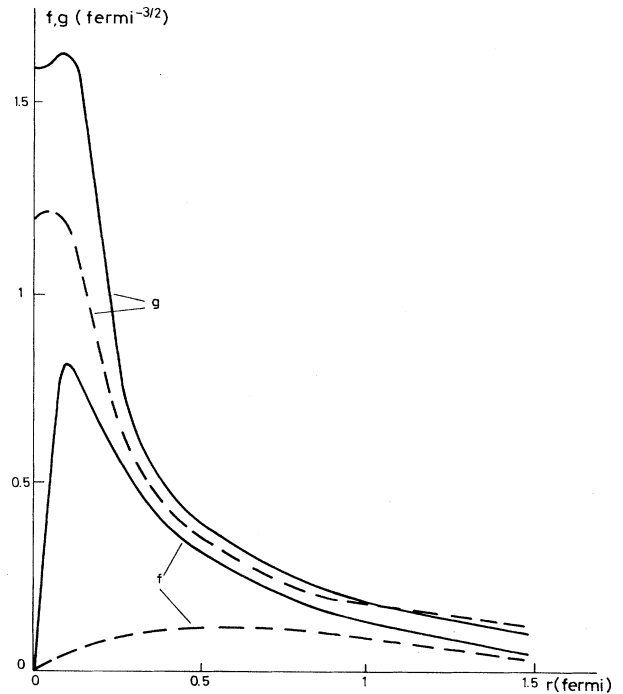


FIG. 2. Radial functions f and g defined as $(f, g) = (m/2\lambda)^{1/2}(F, G)$ vs r in the cases of the baryon or the spin-one meson (solid curve) and the spin-zero meson (dashed curve) in the V model.

tions with nodes was found in spite of a careful search. This contrasts sharply with the Soler model ($z=0$) where there are different branches, one for each value of the number of nodes n , and where there is no more than one solution with the same Ω and n . This follows from numerical calculations and it would be convenient to prove it with analytical methods. To characterize these two situations we will speak of "spiral" and "Sturm-Liouville" behaviors, since the Soler model has a numerable set of solutions, one for each value of the number of nodes, and this recalls the well-known linear Sturm-Liouville systems.

B. Antibaryonlike solutions

There is a charge-conjugate solution to the previous one with $\psi_k=0$, $\phi_k=\phi$, $k=1,2,3$. It has the same energy and spin but opposite baryonic norm as it was explained in I. The fields ϕ have the form (9).

C. Mesonlike solutions

Let us now consider the two-field solutions with zero baryonic norm. They are of the form

$$\psi_i=\psi, \quad \phi_i=\phi, \quad \psi_j=\psi_k=\phi_j=\phi_k=0, \quad (17)$$

where (i,j,k) is any permutation of $(1,2,3)$. As it was shown in I, the condition of vanishing the sources of the four null fields implies that the other two can be written in their most general form as

$$\psi=\psi_1, \quad (18)$$

$$\phi=\cos(\beta/2)\phi_1+e^{i\alpha}\sin(\beta/2)\phi_1,$$

where ψ_1 , ϕ_1 , ϕ_1 are of the forms (4) and (9). As the problem is not separable in spherical coordinates we may substitute (18) in the Lagrangian density and integrate over the angles. The result is

$$L = \frac{4\pi}{\lambda m} \int_0^\infty \left[FG' - GF' - \frac{2}{\rho} FG + \Omega(F^2 + G^2) - (G^2 - F^2) + \frac{\dot{\alpha}}{m}(F^2 + G^2)\sin^2(\beta/2) + \frac{1}{2}(F^2 + G^2) - \frac{4}{9}(F^2 G^2)(1 + 2\cos\beta) \right] \rho^2 d\rho. \quad (19)$$

We now must perform variations of the angles α and β and of the radial functions F and G . The former give

$$\frac{d}{dt}\sin^2(\beta/2)=0, \quad (20)$$

$$\left[\frac{\dot{\alpha}}{m}I_1 + \frac{4}{9}I_3 \right] \sin\beta=0,$$

where I_1 and I_3 are the radial integrals (8). In order to have a stationary state $\dot{\alpha}$ must vanish, from which $\sin\beta=0$ and consequently either $\beta=0$ or $\beta=\pi$, corresponding to parallel and antiparallel spins, respectively, the degeneracy of the model proposed in I being thus bro-

ken. The radial equations for the two cases are then obtained by varying F and G .

1. Parallel spins, $\beta=0$

In this case $\psi=\psi_1$, $\phi=\phi_1$, and the radial equations are of the form (6) with $M=\frac{1}{3}$ corresponding to $z=1$, just the same as for the baryonic solution. Moreover we have

$$E=2 \left[\frac{2\pi}{\lambda m} \right] \mathcal{E}(1), \quad (21)$$

where $\mathcal{E}(1)$ is given by (15) and

$$N=N(\psi)+N(\phi)=0, \quad N(\psi)=\frac{2\pi}{\lambda m^2}I_1, \quad (22)$$

$$\vec{S}=(0,0,N(\psi)).$$

As we see the energy of this solution is $\frac{2}{3}$ times that of the baryon. The baryonic norm is zero, the contributions of ψ and ϕ canceling each other. If m and λ are chosen in such a way that $N(\psi)=\hbar$, the baryon has spin $\frac{3}{2}$ and the meson has spin one.

2. Antiparallel spins, $\beta=\pi$

In this case $\psi=\psi_1$, $\phi=\phi_1$ and we obtain radial equations of the form (6) with $M=-\frac{13}{9}$, corresponding to $z=\frac{11}{3}$. The energy, norm, and spin are, respectively,

$$E=2 \left[\frac{2\pi}{\lambda m} \right] \mathcal{E}\left(\frac{11}{3}\right), \quad (23)$$

$$N=N(\psi)+N(\phi)=0, \quad N(\psi)=\frac{2\pi}{\lambda m^2}I_1,$$

$$\vec{S}=(0,0,0).$$

This represents clearly a spin-zero meson. The search of the solutions of the radial equations was performed and it was found that the behavior is spiral as in the case of the baryon or the spin-one meson. In Fig. 1 we can see the curve $\mathcal{E}(\Omega)$ which has the lowest minimum at $\Omega=0.972$, the center of the spiral being at $\Omega=0.984$, $\mathcal{E}=1.5555$. The numerical values of the energy, baryonic norm, and mean square radius are, respectively,

$$E = \frac{4\pi}{\lambda m} 1.0496,$$

$$N(\psi) = -N(\phi) = \frac{2\pi}{\lambda m^2} 0.8998, \quad (24)$$

$$\langle r^2 \rangle^{1/2} = 3.31/m.$$

If we take $m=286$ MeV, $\lambda=6.488/m^2$ the rest energy, baryonic norm, and mean square radius of three solutions are as follows.

Baryon or antibaryon:

$$\begin{aligned} E &= 1200 \text{ MeV}, \quad S = \frac{3}{2}\hbar, \\ N &= \pm 3\hbar, \quad \langle r^2 \rangle^{1/2} = 1.18 \text{ fm}. \end{aligned} \quad (25a)$$

Spin-one meson:

$$\begin{aligned} E &= 800 \text{ MeV}, \quad S = \hbar, \\ N &= 0, \quad N(\psi) = -N(\phi) = \hbar, \\ \langle r^2 \rangle^{1/2} &= 1.18 \text{ fm}. \end{aligned} \quad (25b)$$

Spin-zero meson:

$$\begin{aligned} E &= 582 \text{ MeV}, \quad S = 0, \\ N &= 0, \quad N(\psi) = -N(\phi) = 0.87\hbar, \\ \langle r^2 \rangle^{1/2} &= 2.31 \text{ fm}. \end{aligned} \quad (25c)$$

As we see, the characteristics of the baryon and of the spin-one meson are very similar to those of the $\Delta(\frac{3}{2}^+)$ and the ω (or the ρ^0). On the other hand, the mass of the spin-zero meson is close to that of the η or the average of those of the π^0 , the η , and the η' mesons.

There are also four-field ($2\psi, 2\phi$) mesons. The condition that $\psi_k = \phi_k = 0$ implies that their two sources must vanish. Just as in I this allows to write without loss of generality the other four fields as

$$\begin{aligned} \sqrt{2}\psi_i &= \psi_\uparrow, \\ \sqrt{2}\phi_i &= \cos(\delta_1/2)\phi_\uparrow - e^{-i\gamma_1}\sin(\delta_1/2)\phi_\downarrow, \\ \sqrt{2}\psi_j &= \cos(\beta/2)\psi_\uparrow + e^{i\alpha}\sin(\beta/2)\psi_\downarrow, \\ \sqrt{2}\phi_j &= \cos(\delta_2/2)\phi_\uparrow - e^{-i\gamma_2}\sin(\delta_2/2)\phi_\downarrow, \end{aligned} \quad (26)$$

where (i, j, k) is any permutation of $(1, 2, 3)$. To obtain the variational approximation we must substitute (26) in the Lagrangian density and proceed in the case of the (ψ, ϕ) solutions. After a straightforward calculation the following solutions are found:

(a) $\beta = \delta_1 = \delta_2 = 0$. The four spins are parallel. The radial equations are of the form (6) with $z = 1$. The energy and spin are the same as in the $(\psi_\uparrow, \phi_\uparrow)$ solution. It corresponds therefore to a spin-one meson with a mass of 800 MeV. The fact that the spin is only one is a consequence of the factor $\sqrt{2}$ in (26).

(b) $\beta = 0, \delta_1 = \delta_2 = \pi$. The two ψ fields have spin up and the two ϕ 's have spin down. The radial equations correspond to $z = \frac{11}{3}$. It has the same physical magnitudes as the $(\psi_\uparrow, \phi_\downarrow)$ solution. It represents a spin-zero meson with a mass of 582 MeV.

(c) $\beta = \pi, \delta_1 = 0, \delta_2 = \pi$ and $\beta = \pi, \delta_1 = \pi, \delta_2 = 0$. One ψ has spin up, the other spin down and the same happens to the ϕ 's. The radial equations have $z = 3$. It represents a spin-zero meson whose mass has been estimated to be between 500 and 550 MeV.

(d) $\beta = 0, \pi, \delta_1 = 0, \delta_2 = \pi$ and $\beta = 0, \pi, \delta_1 = \pi, \delta_2 = 0$. In this annoying solution three spins are parallel while the fourth one is antiparallel. Although this situation is quite natural in quantum mechanics it is disturbing here since its spin, as calculated from the corresponding current, is

close to $\frac{1}{2}$, due again to the $\sqrt{2}$ in (26). It has a mass in the interval 600–650 MeV.

IV. SCALAR-PSEUDOSCALAR COUPLING (S-P MODEL)

Let us consider another spin-dependent coupling obtained from the primitive Lagrangian (1) by inclusion of terms depending on the pseudoscalar bilinear form P :

$$\begin{aligned} L_2 &= \frac{\lambda}{3} \{ [S^2(\psi) + S^2(\phi) + 4S(\psi)S(\phi)] \\ &\quad - [P^2(\psi) + P^2(\phi) + 4P(\psi)P(\phi)] \}, \end{aligned} \quad (27)$$

remaining L_1 and L_3 the same as in Sec. III. In terms of the field of the hadron L_2 can be written as

$$\begin{aligned} L_2 &= \frac{\lambda}{3} [(\bar{\Psi}\Psi)^2 - (\bar{\Psi}\Delta^5\Psi)^2 + 2(\bar{\Psi}P_1\Psi)(\Psi P_2\Psi) \\ &\quad - 2(\bar{\Psi}P_1^5\Psi)(\bar{\Psi}P_2^5\Psi)], \end{aligned} \quad (28)$$

where

$$\Delta^5 = \gamma^5 \otimes I_6, \quad P_1^5 = \gamma^5 \otimes P_1, \quad P_2^5 = \gamma^5 \otimes P_2.$$

The resolution of the corresponding equations can be made along the same lines as in the V model and we will only give the results.

A. Baryonlike solutions

We have again $\psi_k = \psi, \phi_k = 0, k = 1, 2, 3$. The radial equations are of the form (6) with $M = \frac{5}{3}$ corresponding to $z = -1$. There are several branches of solutions characterized everywhere, as in the Soler model, by their number n of nodes, a situation which was referred to above as "Sturm-Liouville behavior." The ground-state nodeless family has the minimum of the energy $\mathcal{E}(\Omega)$ at $\Omega = 0.958$, the physical quantities having the values

$$\begin{aligned} E &= 3 \left[\frac{2\pi}{\lambda m} \right] 4.6096, \\ N &= 3 \left[\frac{2\pi}{\lambda m^2} \right] 4.5314, \\ \langle r^2 \rangle^{1/2} &= 4.06/m, \end{aligned} \quad (29)$$

and the spin is $\frac{3}{2}$.

B. Antibaryonlike solutions

There is a charge-conjugate solution to the previous one with $\psi_k = 0, \phi_k = \phi, k = 1, 2, 3$ with opposite baryonic norm and the same mass and spin.

C. Mesonlike solutions (ψ, ϕ)

1. Parallel spins

The radial equations are the same as in the baryon case ($z = -1$). The energy, norm, and mean square radius are

$$E = 2 \left[\frac{2\pi}{\lambda m} \right] 4.6096 ,$$

$$N = N(\psi) + N(\phi) = 0, \quad N(\psi) = \frac{2\pi}{\lambda m^2} 4.5314 , \quad (30)$$

$$\langle r^2 \rangle^{1/2} = 4.06/m .$$

2. Antiparallel spins

The radial equations, which have a coefficient $M = \frac{7}{9}$ corresponding to $z = \frac{1}{3}$, show some curious features. There exists a family of nodeless solutions for $\Omega > 0.55$ which have associate a curve of the energy showing a behavior with characteristics both of the Dirac-Weyl model ($z = 1$) and of the Soler model ($z = 0$). In fact there exists as for the Dirac-Weyl equation, several nodeless solutions for the same value of Ω in the interval $0.55 < \Omega < 0.85$ and there seems to be an infinity of minima of the energy, being the first one at $\Omega = 0.840$ with

$$E = 2 \left[\frac{2\pi}{\lambda m} \right] 3.1830 ,$$

$$N = N(\psi) + N(\phi) = 0, \quad N(\psi) = \frac{2\pi}{\lambda m^2} 2.2117 , \quad (31)$$

$$\langle r^2 \rangle^{1/2} = 1.39/m ,$$

and zero spin. There is also another local minimum localized on the right region of the curve and separated from all the other minima at $\Omega = 0.920$ with

$$E = 2 \left[\frac{2\pi}{\lambda m} \right] 3.4124 . \quad (32)$$

There exists also, as for the Soler equation, a second family of one-node solution for $\Omega > 0.92$.

In Fig. 3 we plot $G(0)$ against Ω for the nodeless solutions. The function $\mathcal{E}(\Omega)$ and $F(\rho), G(\rho)$ are plotted in

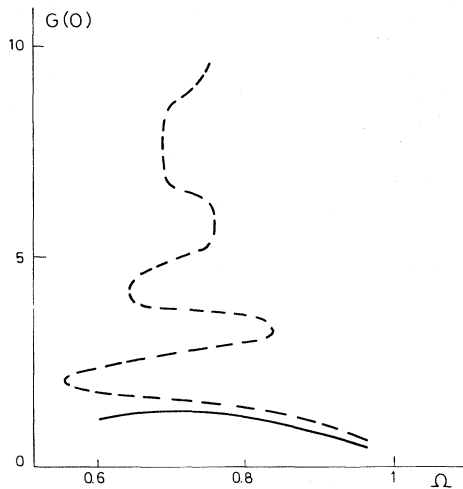


FIG. 3. $G(0)$ as a function of Ω for the nodeless solutions for $z = -1$ (solid curve) and $z = \frac{1}{3}$ (dashed curve).

Figs. 4 and 5. If we take $m = 393$ MeV and $\lambda = 28.47/m^2$ the rest mass E , baryonic norm N , spin S , and mean square radius of the three solutions are the following:

Baryon or antibaryon:

$$E = 1200 \text{ MeV}, \quad S = \left(\frac{3}{2}\right)\hbar , \quad (33a)$$

$$N = \pm \hbar, \quad \langle r^2 \rangle^{1/2} = 2.06 \text{ fm} .$$

Spin-one meson:

$$E = 800 \text{ MeV}, \quad S = \hbar , \quad (33b)$$

$$N = 0, \quad \langle r^2 \rangle^{1/2} = 2.06 \text{ fm} ,$$

$$N(\psi) = -N(\phi) = \hbar .$$

Spin-zero meson:

$$E = 552 \text{ MeV}, \quad S = 0 , \quad (33c)$$

$$N = 0, \quad N(\psi) = -N(\phi) = 0.73\hbar ,$$

$$\langle r^2 \rangle^{1/2} = 0.70 \text{ fm} .$$

As we see the results are very similar to those obtained with the vector coupling.

There are also four-field solutions ($2\psi, 2\phi$) with the form (26). The same values of $\beta, \delta_1, \delta_2$ are also obtained.

(a) $\beta = \delta_1 = \delta_2 = 0$. This has the same mass and spin as the (ψ_1, ψ_1) state, as it corresponds to $z = -1$.

(b) $\beta = 0, \delta_1 = \delta_2 = \pi$. This has the same mass and spin as the (ψ_1, ψ_1) as it corresponds to $z = \frac{1}{3}$.

(c) $\beta = \pi, \delta_1 = 0, \delta_2 = \pi$ and $\beta = \pi, \delta_1 = \pi, \delta_2 = 0$. This has zero spin and a mass of 650 MeV ($z = 0$).

(d) $\beta = 0, \pi, \delta_1 = \delta_2 = 0$ and $\beta = 0, \pi, \delta_1 = \pi, \delta_2 = \pi$. This

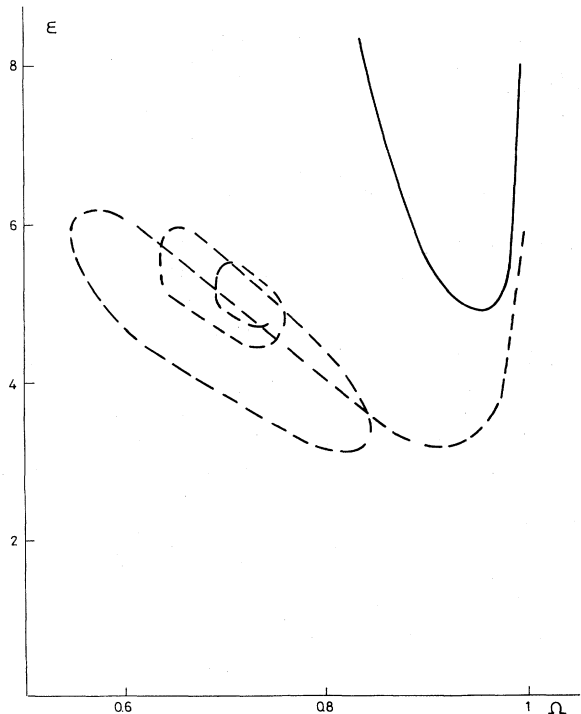


FIG. 4. Same as Fig. 1 but in the S - P model.

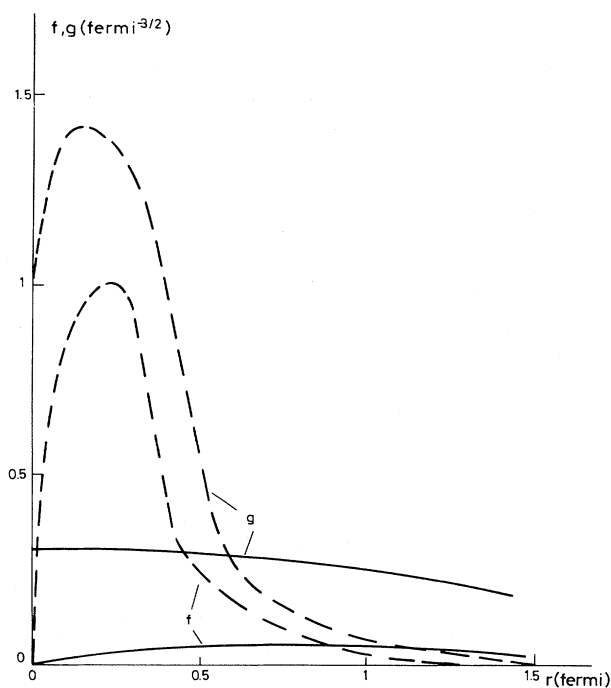


FIG. 5. Same as Fig. 2 but in the S - P model.

annoying solution has a mass slightly greater than 650 MeV but a spin of about $\frac{1}{2}$ cannot have analog in the usual schemes, just as in the case of the V model.

With this coupling we have always found several branches of solutions with different number of nodes or, in other words, what we have called a "Sturm-Liouville behavior." The behavior is spiral, on the other hand, in the case of the vector coupling. The limit between these two types of dependence on Ω of the solutions is located at some value of z in the interval $(\frac{1}{3}, 1)$.

V. NONLINEAR EFFECTS AND THE STRONG INTERACTIONS

As was stressed in I, this model does not make use of gluons or of any other intermediate particle. The forces between the constituents of a hadron are provided by the nonlinear coupling between the different individual Dirac fields, including nonlinear self-actions. Consequently these quarks can be considered to be self-adherent. This suggests that the strong interactions could be considered, at least from a phenomenological point of view, as the effect of the nonlinear superposition of the fields, in a way similar to the interactions between the solitons in the sine-Gordon, KdV , or other analogous equations. This point of view is completely different to the usual one which is mainly based on the study of the forces by means of scattering processes. The present model is very difficult to apply to such problems but, as is shown in this paper, it can be used to obtain the solutions in the rest frame of the particles and to study therefore their spectroscopic properties. We will consider now some arguments which indicate that it is worthwhile to study the nonlinear super-

position of solitary waves as a model for the strong interactions. These are the following.

(i) This nonlinear superposition is very strong, as λm^2 , the dimensionless parameter which characterizes it, is big. Its value is about 23, 6.5, 28.5, respectively, in the models S of I , V , and S - P . It also produces a short-range interaction since the fields decrease exponentially outside of a sphere with a radius of the order of 1 fm.

(ii) The nonlinear superposition can be approximated by the effect of a sum of Yukawa potentials at distances greater than the dimensions of the radius of the solitary waves as was shown a long time ago by Rosen and Rosenstock.⁸ This is a very interesting property since this kind of potential is usually associated with the exchange of particles.

(iii) The present ideas on the structure of hadrons assume that they are composite systems of pointlike constituents which are very loosely bound. Our model does not make use of pointlike entities but of extended quarks. However, and we believe that this is a very important property, their interactions are very slight as long as they are in the ground state, where they are loosely bound. On the other hand, it is very strong if they are pushed apart since they cannot be separated in any way.

In order to evaluate the looseness we may use the two quantities A and B defined as

$$A = \frac{E_{NL}}{E}, \quad B = \frac{E}{mNn} - 1, \quad (34)$$

or

$$A = \frac{\frac{1}{2}I_2 + \frac{2}{3}zI_3}{\Omega I_1 + \frac{1}{2}I_2 + \frac{2}{3}zI_3}, \quad (35)$$

$$B = \frac{(\Omega - 1)I_1 + \frac{1}{2}I_2 + \frac{2}{3}zI_3}{I_1},$$

where E_{NL} is the part of the energy which comes from the fourth-order terms in the energy-momentum tensor and N is the absolute value of the norm of each of the n constituents. A gives the quotient between the binding and the total energies. B gives the energy per unit of mass parameter, norm, and number of constituents minus one and can be interpreted as a measure of the mass renormalization by the nonlinearity.

If A or B are large the systems are very bound or the interactions very strong. Conversely, low values of A and B indicate a slight interaction. In Table I we give the values of A and B in the three models (S , V , S - P).

As we see in two of the cases A and B take surprisingly small values for such an intense interaction. Only in the S - P model their values are remarkable. This paradoxical

TABLE I. Values of A and B in the three models (S , V , S - P).

	S model		V model		S - P model	
	A	B	A	B	A	B
Baryon or spin-one meson	0.088	0.026	0.357	0.399	0.058	0.016
Spin-zero meson	0.088	0.026	0.166	0.166	0.416	0.439

compatibility of low binding energy and very intense interaction is a characteristic of our model which is assumed to hold also in the case of the strong interactions. Nevertheless this point of view meets with a very difficult problem. Granted that the solitary waves represent concentrations of energy which move and can manifest as particles: How do we know that after a collision we have again concentrations of energy? In principle there is no guarantee that a dissipative process does not take place in such a way that the particles disappear, the fields going to zero at every point. This very difficult problem is beyond the scope of this paper. However, we can say that it has been shown that the spinorial solitary waves are very stable under deformations⁹⁻¹¹ in sharp contrast with the scalar case.¹² This makes plausible that after a collision of Dirac solitary waves, the same ones or others emerge as stable final states. In our opinion this is a most important problem which must be studied with the help of analytical techniques and of numerical methods.

To sum up, we believe that it is worthwhile to consider the possibility that the strong interactions could be represented as the effect of the nonlinear superposition of solitary waves, much in the same way as it happens in the case of the solitons.

VI. SUMMARY AND CONCLUSIONS

We have shown two versions of our model of nonlinear extended Dirac quarks corresponding to the basic interactions

$$\bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi, (\bar{\psi}\psi)^2 - (\bar{\psi}\gamma^5\psi)^2,$$

TABLE II. Results in S , V , and S - P models.

	S model	V model	S - P model
m (MeV)	390	286	393
λm^2	22.98	6.488	28.47
Mass of the baryon (MeV)	1200	1200	1200
Mass of the spin-one meson (MeV)	800	800	800
Mass of the spin-zero meson (MeV)	800	582	552

which we call V and S - P models. The most relevant results, together with those obtained with the previous S model, are summarized in Table II. As we see the spin-dependent forces of the V and S - P models break the degeneracy of the mesonic states. The values of the mass parameter are reasonable and the masses of the particles are close to those of the $\Delta(\frac{3}{2}^+)$, the ω , and the η . In order to evaluate this result it must be recalled that these masses have been calculated in the lowest wave approximation. In other words some contribution from higher waves is to be expected.

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