Book of Abstracts ICCSE2 - 27th International Conference on Transport Theory, Bertinoro (Forli, Italy), 10-16 July 2022, p15

Kinetic models for angiogenesis: analysis and simulation

Ana Carpio, Universidad Complutense de Madrid
Elena Cebrián, Universidad de Burgos
Gema Duro, Universidad Autónoma de Madrid

Angiogenesis (growth of blood vessels) is fundamental for tissue development and repair. Numerous inflammatory, immune and malignant diseases are fostered by angiogenic disorders. Hypoxy induced angiogenesis processes including the effect of stochastic motion and branching of blood vessels can be described by an integrodifferential kinetic equation of Fokker–Planck type with source terms that are nonlocal in time, coupled with a diffusion equation for the angiogenic factor [1]. Such models admit soliton-like asymptotic solutions representing the advance of the blood vessels towards the hypoxic regions, such as tumors [2]. We establish the well posedness of the model in the whole space by first constructing the Green functions (fundamental solutions) for the underlying transport problems with variable sources and establishing estimates for their key decay properties [3, 7]. We then implement an iterative linearized scheme whose solutions converge to solutions of the original model, as it follows from comparison principles, sharp estimates of the velocity integrals and compactness results for this type of kinetic and parabolic operators. In bounded domains, the kinetic equations are supplemented with nonlocal boundary conditions and coupled to a diffusion problem with Neumann boundary conditions through the force field created by the tumor induced angiogenic factor and the flux of vessel tips [4]. Lacking explicit expressions for the Green functions, well posedness results exploit balance equations, estimates of velocity decay and compactness results for kinetic operators, combined with gradient estimates of heat kernels for Neumann problems. Our well posedness studies underline the importance of preserving positivity in the schemes employed to approximate numerically solutions. We are able to construct numerical solutions devising order one positivity preserving schemes and show that soliton-like asymptotic solutions are correctly captured [5]. We also find good agreement with the original stochastic model from which the deterministic kinetic equations are derived working with ensemble averages. Higher order positiviy preserving schemes can be devised comibining WENO (weighted essentially non oscillatory) and SSP (strong

stability preserving) discretizations [6].

References

[1] F. Terragni, M. Carretero, V. Capasso, L. L. Bonilla, Stochastic model of tumor-induced angiogenesis: ensemble averages and deterministic equations, Phys. Rev. 93, 022413, 2016

[2] L. L. Bonilla, M. Carretero, F. Terragni, Solitonlike attractor for blood vessel tip density in angiogenesis, Phys. Rev. 94, 062415, 2016

[3] A. Carpio, G. Duro, Well posedness of an integrodifferential kinetic model of Fokker-Planck type for angiogenesis, Nonlinear Analysis-Real World Applications 30, 184-212, 2016

[4] A. Carpio, G. Duro, M. Negreanu, Constructing solutions for a kinetic model of angiogenesis in annular domains, Applied Mathematical Modelling 45, 303-322, 2017

[5] L.L. Bonilla, A. Carpio, M. Carretero, G. Duro, M. Negreanu, F. Terragni, A convergent numerical scheme for integrodifferential kinetic models of angiogenesis, Journal of Computational Physics 375, 1270-1294, 2018

[6] A. Carpio, E. Cebrián, Positivity preserving high order schemes for kinetic models of angiogenesis, International Journal on Nonlinear Sciences and Numerical Simulation, 2022, to appear, https://doi.org/10.1515/ijnsns-2021-0112

[7] A. Carpio, Long-time behavior for solutions of the Vlasov-Poisson-Fokker-Planck equation, Mathematical Methods in the Applied Sciences 21(11), 985-1014, 1998