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A Varma Approach for Estimating Term Premia: the Case of the Spanish Interbank Money Market

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A VARMA APPROACH FOR ESTIMATING TERM PREMIA:

THE CASE OF THE SPANISH INTERBANK MONEY MARKET

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ABSTRACT

This paper highlights the shortcomings of the standard approach of estimating risk premia in the term structure of interest rates. In order to overcome these limitations, a VARMA model based approach is proposed. This procedure is illustrated with the estimation of the term premium implicit in the 30-day interest rate with regard to the 15-day rate, in the Spanish interbank money market.

RESUMEN

En este trabajo se ponen de manifiesto las limitaciones de los métodos tradicionales de las primas por plazo, dentro de la estructura temporal de tipos de interés. Con objeto de solucionarlas se propone un método basado en la estimación de modelos VARMA. Este procedimiento se ilustra con la estimación de la prima por plazo implícita en el tipo de interés a 30 días respecto al tipo a 15 días, en el mercado interbancario español.

Key words: Determination of Interest Rates; Term Structure of Interest Rates, Multiple Time Series Models, Finantial Markets and Macroeconomy.

JEL Classification: E43, C32, E44.

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I. Introduction

The standard solution to the problem of estimating a risk premium in the term structure of interest rates embodies, first, the assumption of a behaviourial equation for the premium, second, the estimation of its relevant parameters, and third, the use of the estimated equation in order to evaluate the premium. Some examples of the mentioned approach are: Jones and Roley(1983), Mankiw and Summers(1984), and Engle, Lilien and Robins(1987).

In these papers, the term premium is assumed to be a linear and static function of some variables in the agents (or researcher) information set. Also, the term premium is not allowed to cause, in the Granger's sense, any of its assumed explanatory variables. Finally, this approach ignores the existence of dynamic relationships among the explanatory variables.

In this paper we show that these assumptions are not compatible with the likely presence of dynamic relationships among the variables included in the researcher information set, and in particular among interest rates. We show that if these dynamic relationships do exist, the term premia will depend on the present and past values of all the variables in the information set, or equivalently, on the present and past innovations associated to all the variables in the information set.

In order to overcome the limitations of the standard approach, a VARMA model based approach to estimating term premia is proposed. This method is illustrated with the estimation of some of the term premia in the Spanish interbank money market. Previous work about term premia estimation in the Spanish interbank money market include Ayuso and De la Torre(1991) and Freixas and Novales(1992).

The remaining of the paper is organized as follows. Section II summarizes the standard procedure for estimating term premia in the term structure of interest rates. Section III derives general analytical expressions for term premia when agents' expectations are based on the present and past history of the relevant set of variables. After that the proposed VARMA model based method for estimating term premia is described. Section IV shows, as an illustration of the method proposed in section III, the estimation of some term premia in the Spanish interbank money market. Finally, section V concludes.

II. Estimating Term Premia

To simplify the exposition, let us assume the existence of two assets, A and B. Maturities are one and two periods respectively, with r_t and R_t being their yearly continuous interest rates.

The term premium implicit in B with regard to A is defined by:

$$\pi_t^{2,1}=2R_t-r_t-E_t(r_{t+1}) \\ =f_{t,t+1}-E_t(r_{t+1}) \qquad (1)$$

where $f_{t,t+1}$ is the forward rate and $E_t(.)$ means the conditional expectation based on information at time t.

The standard method for estimating $\pi_t^{2,1}$ operates as follows. Under the hypothesis that agents' expectations are rational, the relevant parameters of a behaviourial equation for the term premium can be estimated by using one the following models:

a) Jones and Roley(1983)

$$2R_t-r_{t+1}=\pi_t^{2,1}+\beta r_t+\epsilon_{t+1} \\ \pi_t^{2,1}=X_t'\alpha \qquad (2)$$

where α is a vector of parameters and X'_t is a row vector of explanatory variables (U.S. six-month Treasury bill yield, unemployment rate, risk, U.S. Treasury bill supplies and foreign holdings of U.S. Treasury securities). In this formulation $\beta=1$ is an hypothesis to test.

b) Engle, Lilien and Robins(1987)

$$f_{t,t+1}-r_{t+1}=\pi_t^{2,1}+\epsilon_{t+1} \\ \pi_t^{2,1}=\alpha_1+\alpha_2\ln(h_{t+1})+\alpha_3(R_t-r_t) \qquad (3) \\ h_{t+1}^2=\beta_1+\beta_2\sum_{i=1}^{p}\omega_i\epsilon_{t+1-i}^2$$

where $\ln(h_{t+1})$ is a measure of risk, defined as the logarithm of the error term conditional standard deviation.

c) Freixas and Novales(1992)

$$r_{t+1}-r_t=\pi_t^{2,1}+\beta(f_{t,t+1}-r_t)+\epsilon_{t+1} \qquad (4) \\ \pi_t^{2,1}=\alpha_1+\alpha_2 v_t$$

where v_t is the short term interest rate volatility, defined as in Fama(1976).

In these three cases, a consistent estimate of $\pi_t^{2,1}$ can be obtained by estimating any of the following vectors of parameters: $(\beta\ \alpha)$, $(\alpha_1\ \alpha_2\ \alpha_3\ \beta_1\ \beta_2\ \omega_1\ \dots\ \omega_p)$ or $(\beta\ \alpha_1\ \alpha_2)$. While this approach avoids computing $E_t(r_{t+1})$ in the process of estimating $\pi_t^{2,1}$, it introduces a new and arbitrary element, i.e. the unidirectional static behaviourial equation for $\pi_t^{2,1}$. Thus, models (2) - (4) lead to the following questions: (1) why should the relationship between $\pi_t^{2,1}$ and the components of X'_t be static?, (2) should not we allow for feedback relationships?, and (3) should not we take into account the dynamic relationships among the components of X'_t in order to improve term premia estimation?. In spite of their relevance, and to the best of our knowledge, these constraints have never been checked in the empirical literature on term premia estimation.

III. A VARMA Approach for Term Premia Estimation

In this section, analytical expressions for the term premium are derived. They are based on the assumption that the relevant variables in the information set follow a general non-stationary VARMA process.

For simplicity, the information set held by the agents is assumed to contain the present and past values of a 4×1 vector, z_t, of variables. The short term interest rate, r_t, and the long term interest rate, R_t, together with two any other variables related to r_t and R_t, namely, x_t and y_t.

Let's assume that z_t follows the process:

$$z_t=\Psi(B)e_t \tag{5}$$

where e_t is a vector of independent, identically and normally distributed random variables, with contemporaneous covariance matrix Σ and $\Psi(B)$ being an infinite order polynomial matrix in B, the back-shift operator, normalized so that $\Psi(0)=I$. Hence, the generic element for $\Psi(B)$ takes the form:

$$\begin{aligned}\psi_{i,j}(B)&=1+\psi_{ij,1}B+\psi_{ij,2}B^2+\psi_{ij,3}B^3+\dots \qquad for\ i=j\\ &=\psi_{ij,1}B+\psi_{ij,2}B^2+\psi_{ij,3}B^3+\dots \qquad for\ i\neq j\end{aligned} \tag{6}$$

If the variables in z_t are integrated of order 1 with no cointegrating relationships, $\Psi(B)$ can be factorized as:

$$\begin{aligned}&\Psi(B)=D^{-1}\Psi^*(B)\\ &D^{-1}=\nabla^{-1}I_{(4x4)}\\ &\Psi^*(B)=\Phi^{-1}(B)\Theta(B)\end{aligned} \tag{7}$$

where the roots of $|\Phi(B)|=0$ and $|\Theta(B)|=0$ lie outside the unit circle. In this

case a VARMA model for ∇z_t can be obtained following Jenkins and Alavi(1981) or Tiao and Box(1981).

If there are "r" cointegration relationships in z_t, the above factorization does not exist. In that case it is possible to define a new 4×1 vector, z^*_t, whose elements are: "r" cointegrating relationships and "4-r" first differenced, independent linear combinations of elements in z_t. A VARMA process for z_t can then be obtained from the VARMA process for z^*_t.

In both cases, the expression relating the term premium $\pi_t^{2,1}$ to the variables in z_t can be obtained from (1) and (5) as[1]:

$$\pi_t^{2,1}=S(B)e_t \tag{8}$$

where

$$e_t=[e_{xt}\ e_{rt}\ e_{Rt}\ e_{yt}]' \tag{9}$$

is the error vector and S(B) is the polynomial row vector

$$S(B)=[S_x(B)\ S_r(B)\ S_R(B)\ S_y(B)] \tag{10}$$

with

$$\begin{aligned}S_x(B)&=[2B\psi_{3,1}(B)-B\psi_{2,1}(B)-\psi_{2,1}(B)]B^{-1}\\ S_r(B)&=[2B\psi_{3,2}(B)-B\psi_{2,2}(B)-\psi_{2,2}(B)+1]B^{-1}\\ S_R(B)&=[2B\psi_{3,3}(B)-B\psi_{2,3}(B)-\psi_{2,3}(B)]B^{-1}\\ S_y(B)&=[2B\psi_{3,4}(B)-B\psi_{2,4}(B)-\psi_{2,4}(B)]B^{-1}\end{aligned} \tag{11}$$

Using (5) and (8), $\pi_t^{2,1}$ can also be represented as:

$$\pi_t^{2,1}=S(B)\Psi^{-1}(B)z_t \tag{12}$$

Equation (8) relates $\pi_t^{2,1}$ to current and past one-step-ahead forecasting errors, corresponding to all variables in z_t. These errors have associated a lag structure in $\pi_t^{2,1}$ given by the components of S(B). Note from (7) that the absence of cointegration implies the elements of S(B) to share a factor ∇^{-1}, i.e. $\pi_t^{2,1}$ will be an I(1) variable[2].

This representation is of particular interest because it gives an intuitive interpretation of the term premium, relating its size to the present and past forecasting errors of the variables in the information set. On one hand, present innovations indicate agents' reactions (changes in the premium) to current events. Its relative weight in the determination of a term premium (measured as its contribution to the total variance) constitutes an indication of the importance given by agents to the unforecastable current events. On the other hand, the presence of past forecasting errors indicates that agents do not adjust immediately their premia. Also, the presence of these past errors means that the term premia are forecastable, i.e. they are not white noise processes.

Equation (12) relates $\pi_t^{2,1}$ to z_t. It is clear that if no cancellations occur in the vector $S(B)\Psi^{-1}(B)$, $\pi_t^{2,1}$ will depend on present and past values of all variables in z_t. Thus, by assuming that $\pi_t^{2,1}$ is a linear and static function of some z_t components, it introduces many a priori zero constraints on the components of $S(B)\Psi^{-1}(B)$.

The above discussion highlights the shortcomings of the standard approach that may lead to inadequate estimations of the term premium. These limitations can be overcomed by estimating the term premium with the following procedure:

1) Obtain the number of cointegration relationships in z_t.
2) Specify a VARMA model for either z_t or z^*_t, depending on the number of cointegrating relationships.
3) Compute S(B) and estimate $\pi_t^{2,1}$ using (8) or (9).

This procedure has the additional advantage that embodies the standard approach as a particular case. Note that (9) might degenerate to:

$$\pi_t^{2,1} = S(0)\Psi^{-1}(0)z_t \qquad \textbf{(13)}$$

implying a purely static relationship.

An important feature of the proposed VARMA framework is that it allows for specification of the diferent set of constrains on the feedback/dynamic relationships among variables in the information set, implied by the relevant theories of the Term Structure of Interest Rates.

Shiller and McCulloch(1987) interpret and consolidate most of the literature

on the Term Structure of Interest Rates; they summarize all different theories along with the available empirical evidence. From this survey, equilibrium models seem to lead to two apparently opposite views about the Term Structure. The first one is linked to hypotheses or assumptions which imply constant or even zero term premia, while the second is related to those implying time varying term premia. By adopting the first view, expectations on future short term rates are able to explain most of the behaviour (up to a constant liquidity or risk premium) of longer term rates. For example, the Log Expectations Hypothesis (LEH) [see McCulloch(1993)] is very informative, since it identifies the main determinants of the longer interest rates stochastic movements. However if a time varying term premium is allowed, the LEH loses part of its explanatory power, unless it is possible to identify the determinants of movements in the premium. In the absence of those determinants, the higher the contribution of the premium in explaining the behaviour of long term interest rates, the bigger the loss in the explanatory power of the LEH.

As we can see from (8), a zero or constant term premium requires $S(B)=0$, which implies a large number of constrains on the dynamics of the VARMA model. For the premium to be I(0), the series of coefficients associated with each lag polynomial in S(B) must converge. This condition is equivalent to the one obtained by Hall, Anderson and Granger(1992). They show that each continuously compounded yield to maturity of a k period pure discount bond ($k=1,2,3,..n$) must be cointegrated with the yield of a 1 period pure discount bond. Moreover, the spreads must be I(0). This implies that in a vector of "n" yields, a number of "n-1" cointegration relationships must be present. In this section we have shown the opposite proposition, i.e. the lack of cointegration among the variables in the information set leads to a I(1) term premium.

The case of I(0) term premia is particularly important because it implies that expectations on future short term interest rates are able to explain (at least) an important feature of longer term rates: the stochastic trend. Thus, although the LEH is rejected because term premia are not constant, it continues to be very informative. In particular, if a I(0) term premium follows a white noise process with a small variance (compared with the variance of the longer term rate) it means that a relaxed version of the LEH could be accepted.

Note that a I(0) premium is not necessarily a white noise variable, i.e. a I(0) variable might show autocorrelation. In that case, past one period ahead forecast errors will form part of the premium. This means that, facing a big forecast error, agents do not fully modify the premium instantaneously, but instead they do it gradually during subsequent periods. The autocorrelation shown by the premium can be interpreted as the likely presence of autocorrelation in expected risk, as implied by a typical consumption based capital asset pricing model, in which a representative agent with risk aversion maximizes his expected utility subject to an intertemporal budget constraint [see for example Engle, Lilien and Robins(1987)].

IV. Empirical Analysis

In order to estimate the term premium implicit in the 30-day Spanish interbank interest rate with regard to the 15-day rate, the multivariate estimation approach proposed in Section III was used.

The information set is assumed to contain the present and past values of the 4×1 vector w_t = $(R1_t\ R7_t\ R15_t\ R30_t)'$ where:

$$\begin{aligned} R1_t &\equiv \ln\left[1+\frac{1}{360}s1_t\right] \\ R7_t &\equiv \ln\left[1+\frac{7}{360}s7_t\right] \\ R15_t &\equiv \ln\left[1+\frac{15}{360}s15_t\right] \\ R30_t &\equiv \ln\left[1+\frac{30}{360}s30_t\right] \end{aligned} \quad \mathbf{(14)}$$

and $s1_t$, $s7_t$, $s15_t$ y $s30_t$ are 360 days basis, simple interest rates, corresponding to 1, 7, 15 and 30 days to maturity, of the Spanish interbank money market. The

variables $R1_t$, $R7_t$, $R15_t$ and $R30_t$ are the logs of the yields to maturity by peseta invested in each type of loan. These variables are directly proportional to the continuously compounded yields to maturity. The exact relationship is:

$$\frac{360}{N}RN_t = r_{N,t} \quad \mathbf{(15)}$$

where r_{Nt} is the continuously compounded N-day yield to maturity, N=1, 7, 15 and 30 days. The sample size was 116 weekly observations, from 4/1/89 to 20/3/91.

Note that z_t and w_t are related though:

$$z_t = \Lambda w_t \quad \mathbf{(16)}$$

where

$$\Lambda = \begin{bmatrix} \frac{360}{1} & 0 & 0 & 0 \\ 0 & \frac{360}{7} & 0 & 0 \\ 0 & 0 & \frac{360}{15} & 0 \\ 0 & 0 & 0 & \frac{360}{30} \end{bmatrix} \quad \mathbf{(17)}$$

For this particular illustration of the term premia estimation procedure proposed in Section III, the information set has been constrained to include only four variables; this assumption keeps the dimension of the problem within reasonable bounds. The inclusion of the 15-day and 30-day interest rates follows from the objective of estimating the term premium implicit in the 30-day interest rate with regard to the 15-day rate. The one-day interest rate has been included because it is an important control variable for the Banco de España. Both, the behaviour of this variable and the expectations on its future values, are believe to be able to explain a good deal of the behaviour of the remaining interbank interest rates. Finally, the seven-day interest rate has been included because it allows to compute other important term premia, i.e. $\pi_t^{15,7}$ and $\pi_t^{30,7}$. To keep the size of the information set within reasonable bounds, we do not include in this analysis other potentially important explanatory variables like volatility. Nevertheless, enlarging of the size of the information set is a natural extension of this exercise.

In estimating $\pi_t^{30,15}$, the 15-day and 30-day interest rates play the role of r_t and R_t (of Section III) respectively. The one-day and seven-day interest rates play the role of x_t and y_t, respectively. When estimating $\pi_t^{15,7}$ and $\pi_t^{30,7}$, these roles would change accordingly.

Table 1 shows a summary of the univariate stochastic (US) models for each variable. The parameters θ_1, θ_2 and θ_3 correspond to a MA(3) process, σ is the residual standard deviation and Q(20) is the Ljung-Box statistic with 20 degrees of freedom.

(Introduce Table 1)

All variables seem to be integrated of order 1. Therefore, before elaborating the VARMA model, an analysis of cointegration is necessary. We use Johansen's(1988) Trace Test of cointegration. This test requires an assumption about "p", i.e. the order of the VAR(p) process which approximates best the generating process for w_t. Table 2 shows the AIC and M(p) statistics for different values of "p". Both indicate that p should be at least 3.

(Introduce Table 2)

Table 3 shows the Trace Test for different values of "p" (a constant term has been present in all regressions). The tested null hypothesis are:

(a) At most three cointegration relationships

(b) At most two cointegration relationships

(c) At most one cointegration relationship

(d) None cointegration relationship

In order to increase the power of the test in short samples we have applied Reimers' transformation [see Banerjee et al.(1993), page 286]. This transformation consist of multiplying the values of the trace statistic times [1-(kp/T)], where k is

the number of variables and T is the effective number of observations.

(Introduce Table 3)

No cointegration relationships appear at 99% if p>3. When p=3 the trace statistic lightly rejects the null hypothesis of zero cointegration relationships[3]. The presence or absence of a cointegration relationship depend on the choice of "p". As the US models in Table 1 indicate that the first difference of every interest rate follows a MA process, this suggests a vector MA process for ∇w_t. This process will be non-invertible if there is a cointegration relationship within w_t. If there is not such a cointegration relationship, a VAR(p) with p>3 will be necessary in order to approximate the VARMA(1,3) for w_t.

We decide to proceed under the assumption of zero cointegration relationships, although we give special attention to potential non-invertibility problems. The estimation algorithm used[4], will fail to reach convergence if non-invertible problems arise.

Using Jenkins and Alavi's(1981) methodology we specified and estimated the following VARMA model:

$$\begin{bmatrix} \nabla R1_t \\ \nabla R7_t \\ \nabla R15_t \\ \nabla R30_t \end{bmatrix} = \begin{bmatrix} \Psi_{1,1}^*(B) & 0 & 0 & \Psi_{1,4}^*(B) \\ \Psi_{2,1}^*(B) & \Psi_{2,2}^*(B) & 0 & \Psi_{2,4}^*(B) \\ \Psi_{3,1}^*(B) & 0 & \Psi_{3,3}^*(B) & \Psi_{3,4}^*(B) \\ 0 & 0 & 0 & \Psi_{4,4}^*(B) \end{bmatrix} \begin{bmatrix} a_{1t} \\ a_{7t} \\ a_{15t} \\ a_{30t} \end{bmatrix} \qquad \textbf{(18)}$$

where:

$$\Psi_{1,1}^*(B)=1-\underset{(.06)}{.57}B+\underset{(.06)}{.20}B^3$$

$$\Psi_{1,4}^*(B)=\underset{(.004)}{.02}B-\underset{(.004)}{.01}B^2$$

$$\Psi_{2,1}^*(B)=\underset{(.32)}{1.21}B^3$$

$$\Psi_{2,2}^*(B)=1-\underset{(.06)}{.42}B-\underset{(.05)}{.10}B^3 \tag{19}$$

$$\Psi_{2,3}^*(B)=\underset{(.02)}{.07}B-\underset{(.01)}{.02}B^2$$

$$\Psi_{3,1}^*(B)=-\underset{(.37)}{1.21}B$$

$$\Psi_{3,3}^*(B)=1$$

$$\Psi_{3,4}^*(B)=-\underset{(.02)}{.05}B^2$$

$$\Psi_{4,4}^*(B)=1$$

Table 4 shows some useful statistics for the residuals of this model. The residual standard deviation (col.2), the Ljung-Box statistic with 20 degrees of freedom (col. 3), residual autocorrelations of orders 1, 2 and 3 (cols. 4-6) and the Ljung-Box statistic, computed on the squared residuals, with 20 degrees of freedom (col. 7).

(Introduce Table 4)

Partial and cross correlation matrices on residuals, up to the 20th lag, were also computed. Those results are not presented here but they did not show any sign of stochastic structure in the residuals.

The model (16)-(17) indicates that w_t follows a VARMA(1,3). The determinant of the polynomial MA matrix has its roots outside the unit circle (this determinant is obtained as the product $\Psi_{1,1}^*(B)\Psi_{2,2}^*(B)$). The estimation algorithm converges quickly, indicating the absence of non-invertibility problems, i.e. the

absence of cointegration relationships, or in other words the absence of a common factor driving the term structure.

From the structure of the model (16)-(17) it follows that interest rates in the Spanish interbank money market are dynamically related, i.e. this market does not fulfil the constrains of the standard approach for estimating term premia. These are I(1) variables and depend on current and past values of the variables in w_t. Also, the particular structure of the model implies that R30 has relevant information in forecasting shorter term rates.

These two last results, i.e. nonstationary term premia and forecastability of shorter term interest rates from longer term ones, are similar to those found by Hall, Anderson and Granger(1992) for the U.S. Treasury bill yields and for the period 1979:10 - 1982:9. During this period, the Federal Reserve ceased targeting interest rates. These authors fail to find stationary term premia although they do find it for periods in which the Federal Reserve targeted interest rates as an instrument of monetary policy. In our sample period the Banco de España did not use interest rates as the target for monetary policy. As in our case these authors find that longer term Treasury bills have useful information in forecasting shorter term bills.

The VARMA(1,3) model can be expressed as:

$$\nabla w_t = \Theta(B)a_t \tag{20}$$

Thus, the model for the vector of continuous interest rates z_t is:

$$\begin{aligned}\nabla z_t &= (\Lambda\Theta(B)\Lambda^{-1})(\Lambda a_t)\\ &= \Psi^*(B)e_t\end{aligned} \tag{21}$$

Note that (21) is equal to (5) with Ψ(B) factorized as in (7).

The term premium $\pi_t^{30,15}$ derived from the estimated version of (21) is:

$$\nabla \pi_t^{30,15} = \frac{360}{15}[(1.28+1.28B)a_{1t} + (1+.05B+.05B^2)a_{30t} - (2)a_{15,t}] \quad (22)$$

Following a similar procedure $\pi_t^{30,7}$ and $\pi_t^{15,7}$ were obtained as:

$$\nabla \pi_t^{30,7} = \frac{360}{7}[(-1.21-1.21B-1.21B^2-1.21B^3)a_{1t} + (-3.64+.52B+.10B^2+.10B^3)a_{7t} + (.83-.05B+.02B^2)a_{30,t}] \quad (23)$$

$$\nabla \pi_t^{15,7} = \frac{360}{7}[(-1.28B-1.21B^2-1.21B^3)a_{1t} + (-1.58+.42B+.20B^2)a_{7t} + (-.07-.05B-.03B^2)a_{30,t} + (1)a_{15,t}] \quad (24)$$

Figures 1, 2 and 3 show the paths followed by these term premia.

(Introduce Figures 1, 2 and 3)

Our three estimated term premia are I(1) variables, and while $\pi_t^{30,15}$ changes sign very often, $\pi_t^{30,7}$ and $\pi_t^{15,7}$ remain positive for the whole sample.

Equations (22)-(24) show the term premia as functions of present and past one-step-ahead forecast errors. Most of these errors have associated a lag structure, implying that:

(1) Increments in term premia are predictable and do not converge instantaneously to their mean value (zero), but rather they need two or three weeks.

(2) The bigger size of the coefficients associated to the <u>current</u> one-step-ahead forecast errors compared with those associated to <u>past</u> errors, show that changes in term premia seem to be explained mainly by the former. In other words, although it takes the agents a few weeks to adjust completely their premia, the biggest part of the adjustment takes place instantaneously (within the week). Therefore, agents seem to give great importance to current surprises when setting their premia.

(3) As can be seen in (22)-(24) some of the surprises are not taken into account in setting some of the premia, for instance, forecasting errors in R7 are not directly taken into account in setting $\pi_t^{30,15}$. The same happens with forecasting errors in R15 when agents are setting $\pi_t^{30,7}$. However, one must be cautious in using (22)-(24) to discuss which surprises are the most relevant in determining the premia behaviour. Forecast errors in (22)-(24) are not independent, they show high contemporaneous correlation that makes impossible to separate their specific contributions.

These contemporaneous correlations among errors can be interpreted as within-week effects among interest rates. Assuming a specific set of within-week relationships, leads to a particular orthogonalization of these errors, and that to a particular set of specific contributions. Thus, the relative importance of a specific variable in determining the behaviour of the term premium will depend on the specific within-week relationships assumed. That kind of analysis is out of the scope of this paper and constitutes one of its natural extensions.

V. Conclusions

This paper deals with the problem of estimating term premia in the term structure of interest rates. It has been proved that the standard approach, based on static specifications of behaviourial equations for term premia, is not consistent with the presence of dynamics among interest rates at different maturities. So, that approach may lead to inappropriate estimations of term premia.

In order to overcome the limitations of the standard analysis a multivariate stochastic approach is proposed.

When this method is applied to the Spanish interbank money market, important features of its term structure arise. Our empirical analysis shows that:

1) Interest rates in the Spanish interbank money market are dynamically related, against the standard assumption. This leads term premia to depend on

current and past values of the variables in the information set.

2) The 30-day interest rate seems to have useful information in forecasting shorter term interest rates. The latter do not seem to have much information in order to forecast the former. A similar result has been found by Hall, Anderson and Granger(1992) for U.S. Treasury bill yields.

3) The weekly time series for the different rates analyzed here, do not seem to be cointegrated, implying nonstationary term premia (at least during the sample period considered). Hall, Anderson and Granger(1992) find a similar result for U.S. Treasury bill yields for a period in which the Federal Reserve did not target interest rates. Our sample period shares the same feature, i.e. the Banco de España did not use interest rates as a target. This result suggests that by controlling short term interest rates the Banco de España has not full control on longer term interest rates, since all term premia are nonstationary I(1) variables. Also, this means that the Log Expectations Hypothesis (even in its more relaxed version) is not very informative, since the behaviour of longer term interest rates is mainly explained by their corresponding premia. Agents do not adjust premia instantaneously and premia changes are predictable. These changes depend on current and past forecast errors associated to variables in the information set, with the former having a larger weight. This implies that in order to explain the behaviour of premia changes it will be necessary to explain the one step ahead forecast errors of those variables.

Finally, our results indicate that, during the period considered, expectations on shorter term interest rates have not been very helpful in explaining the behaviour of longer term interest rates, since these expectations have not been able to explain the most evident feature of longer term rates: their stochastic trend. If, as usually thought, spending decisions and capital-asset valuations depend primarily on long-term rates, our results could cast doubt about the effectiveness of a monetary policy based on the control of short term interest rates. In this sense, two important questions arise: (1) will our results hold when a short interest rate and an actually longer term interest rate are considered? and (2) by targeting short term interest rates, will the monetary authority be able to affect aggregate demand?. Given the short period considered in our empirical example as well as the special characteristics of the interbank money market, it would be very risky to infer any

answer to these two important questions. Agents in this market (banks) are constrained to observe strict legal regulations (e.g. a cash coefficient each 10 days). This fact might lead them to behave, to some extend, along the lines of the Preferred Habitat Hypothesis (Modigliani and Sutch,1966) in which movements in interest rates may be mainly determined by heterogeneous liquidity positions of agents.

The expansion of the information set by incorporating long term interest rates, aggregate demand variables and different measures of risk, are natural extensions of this paper.

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APPENDIX

Consider the vector $z_t = (x_t\ r_t\ R_t\ y_t)'$ which follows the process:

$$\begin{bmatrix} x_t \\ r_t \\ R_t \\ y_t \end{bmatrix} = \begin{bmatrix} \Psi_{1,1}(B) & \Psi_{1,2}(B) & \Psi_{1,3}(B) & \Psi_{1,4}(B) \\ \Psi_{2,1}(B) & \Psi_{2,2}(B) & \Psi_{2,3}(B) & \Psi_{2,4}(B) \\ \Psi_{3,1}(B) & \Psi_{3,2}(B) & \Psi_{3,3}(B) & \Psi_{3,4}(B) \\ \Psi_{4,1}(B) & \Psi_{4,2}(B) & \Psi_{4,3}(B) & \Psi_{4,4}(B) \end{bmatrix} \begin{bmatrix} e_{xt} \\ e_{rt} \\ e_{Rt} \\ e_{yt} \end{bmatrix} \qquad \textbf{(25)}$$

From (25):

$$2R_t = 2\Psi_{3,1}(B)\ e_{xt} + 2\Psi_{3,2}(B)\ e_{r,t} + 2\Psi_{3,3}(B)\ e_{Rt} + 2\Psi_{3,4}(B)\ e_{yt}$$

$$r_t = \Psi_{2,1}(B)\ e_{xt} + \Psi_{2,2}(B)\ e_{r,t} + \Psi_{2,3}(B)\ e_{Rt} + \Psi_{2,4}(B)\ e_{yt}$$

$$E_t(r_{t+1}) = \Psi_{2,1}(B)B^{-1}e_{xt} + (\Psi_{2,2}(B)-1)B^{-1}e_{r,t} + \Psi_{2,3}(B)B^{-1}e_{Rt} + \Psi_{2,4}(B)B^{-1}e_{yt}$$

Since

$$\pi_t^{2,1} = 2R_t - r_t - E_t(r_{t+1})$$

the term premium can be represented as:

$$\pi_t^{2,1} = S(B)e_t \qquad \textbf{(8)}$$

where

$$S(B) = [S_x(B)\ S_r(B)\ S_R(B)\ S_y(B)] \qquad \textbf{(10)}$$

with

$$\begin{aligned} S_x(B) &= [2B\psi_{3,1}(B) - B\psi_{2,1}(B) - \psi_{2,1}(B)]B^{-1} \\ S_r(B) &= [2B\psi_{3,2}(B) - B\psi_{2,2}(B) - \psi_{2,2}(B) + 1]B^{-1} \\ S_R(B) &= [2B\psi_{3,3}(B) - B\psi_{2,3}(B) - \psi_{2,3}(B)]B^{-1} \\ S_y(B) &= [2B\psi_{3,4}(B) - B\psi_{2,4}(B) - \psi_{2,4}(B)]B^{-1} \end{aligned} \qquad \textbf{(11)}$$

FOOTNOTES

1. See appendix for mathematical details
2. Hall, Anderson and Granger(1992) show that two I(1) interest rates are cointegrated if and only if the term premium is I(0).
3. The λ-max test statistic takes the value of 31.3, as the 99% critical value is 33.2, the null hypothesis of zero cointegration relationships is not rejected.
4. The SCA Statistical System was used in order to carry out the computations. This software uses an exact maximum likelihood estimation algorithm based on Hillmer and Tiao(1979).

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I want to thank E. Domínguez, M. Gracia, A. Novales, T. Pérez and an anonymous referee for their valuable comments and suggestions. I am responsible for all remaining errors. Financial support is also acknowledged from the DGICYT project PB93-1277.

Table 1: Univariate Models

	θ_1	θ_2	θ_3	σ%	Q(20)
$\nabla R1_t$	.37 (.10)	.04 (.10)	.20 (.10)	.0008	15.6
$\nabla R7_t$	.20 (.10)	-	-	.0042	18.1
$\nabla R15_t$	-.15 (.09)	-.03 (.09)	-.23 (.09)	.0081	21.0
$\nabla R30_t$	-	-	-	.0142	17.5

Table 2: M(p) and AIC Statistics for Different Lags

p	1	2	3	4	5	6	7	8	9	10	11	12
M(l)	234.4	29.7	38.4	7.5	17.6	13.6	18.4	9.9	18.3	10.2	18.2	26.2
AIC	-87.0	-87.1	-87.3	-87.1	-87.0	-86.9	-86.9	-86.8	-86.8	-86.7	-86.7	-87.0

Note: The 97.5% critical value for M(p) is 28.8

Table 3: Trace Statistic for Different Lags

p	3	4	5	6	7	8	9	10	11	12	C.V.	
H_0											97.5%	99%
3	.2	.0	.2	.4	.5	.1	.1	.1	.2	.0	10.8	13.0
2	10.9	10.7	7.3	8.4	6.7	4.5	3.8	3.1	4.5	2.9	22.1	24.6
1	29.3	27.2	25.7	21.7	20.4	13.5	11.3	10.6	12.1	7.7	37.6	41.1
0	60.6	52.9	57.1	47.8	44.9	36.0	28.1	27.3	29.5	27.0	56.1	60.2

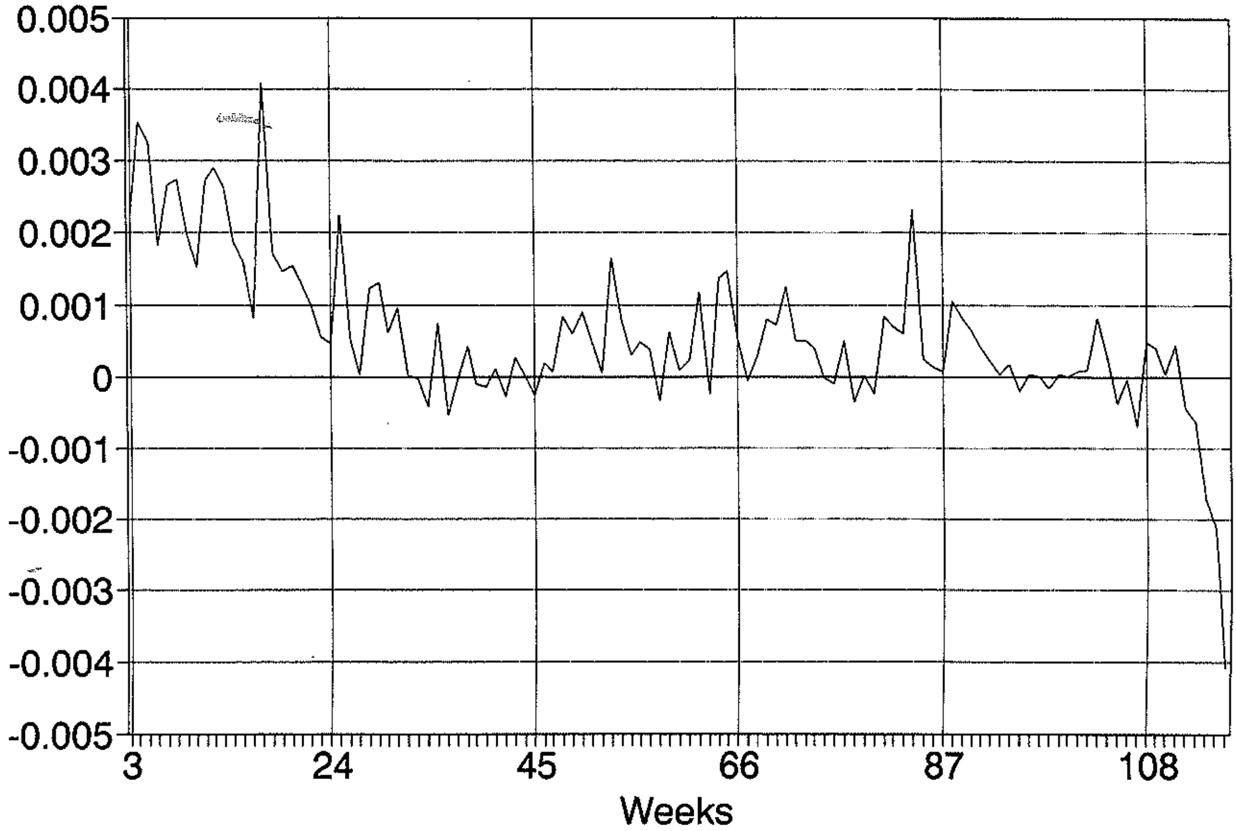
Note: C.V. are the critical values in Osterwald-Lenum(1992)

Table 4: Some Statistics on Residuals

	σ%	Q(20)	ρ_1	ρ_2	ρ_3	Q^*
$\nabla R1_t$	.0008	13.4	.00	.05	.03	6.2
$\nabla R7_t$	.0040	16.6	.12	.02	.03	17.6
$\nabla R15_t$	.0080	21.7	.21	.10	.18	23.8
$\nabla R30_t$	.0142	17.8	.21	.00	-.01	15.7

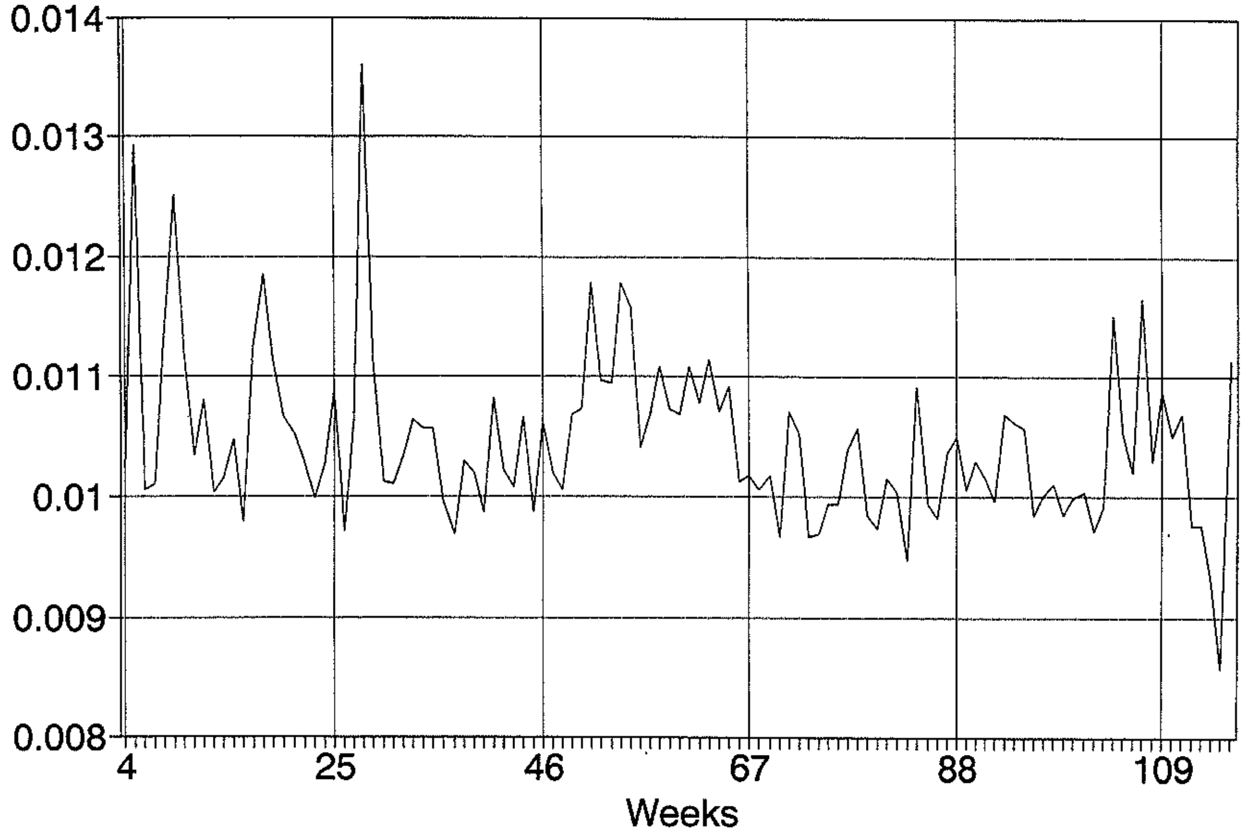
TERM PREMIUM

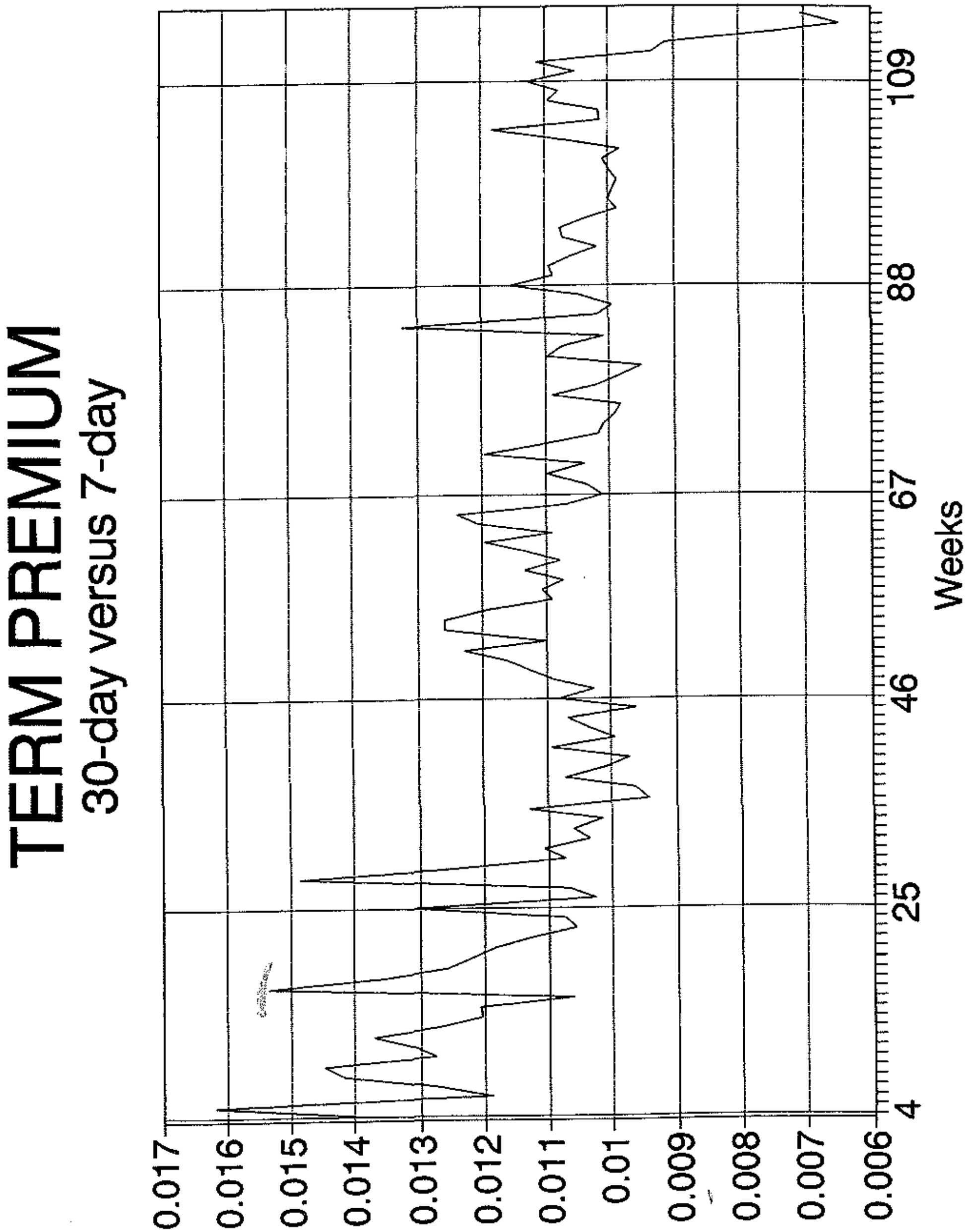
30-day versus 15-day



TERM PREMIUM

15-day versus 7-day





DATA SET

R1	R7	R15	R30
0.00035	0.002479	0.005383	0.010902
0.000337	0.002420	0.005249	0.010874
0.000353	0.002472	0.005370	0.010926
0.000343	0.002408	0.005243	0.01078
0.000336	0.002503	0.005545	0.011362
0.000372	0.002621	0.005661	0.011474
0.000373	0.002617	0.005655	0.011532
0.000372	0.002628	0.005731	0.011691
0.000373	0.002665	0.005851	0.011867
0.000388	0.002734	0.005935	0.011998
0.000388	0.002724	0.005879	0.011988
0.00039	0.002709	0.005868	0.011978
0.000386	0.002713	0.005844	0.011909
0.000383	0.002681	0.005785	0.011727
0.00038	0.002703	0.005842	0.011817
0.000392	0.002741	0.005890	0.011848
0.000377	0.002671	0.005811	0.011962
0.000399	0.002763	0.006020	0.012183
0.000386	0.002732	0.005927	0.011977
0.000384	0.002696	0.005836	0.011802
0.000387	0.002713	0.005864	0.011837
0.000391	0.002747	0.005922	0.011927
0.000393	0.002757	0.005930	0.011908
0.000402	0.002820	0.006068	0.012176
0.000383	0.002723	0.005899	0.011985
0.000422	0.002938	0.006281	0.012607
0.000433	0.003022	0.006487	0.012978
0.000401	0.002887	0.006341	0.012785
0.000421	0.002969	0.006402	0.012914
0.000423	0.002966	0.006354	0.01276
0.000408	0.002918	0.006257	0.012594
0.000422	0.002967	0.006365	0.012731
0.000423	0.002967	0.006377	0.012753
0.000421	0.002947	0.006334	0.012634
0.000414	0.002912	0.006264	0.01259
0.000425	0.002967	0.006349	0.012654
0.000422	0.002946	0.006296	0.012588
0.000408	0.002867	0.006163	0.012362
0.000411	0.002876	0.006177	0.012345
0.000413	0.002889	0.006189	0.012367
0.000407	0.002872	0.006195	0.0124
0.000412	0.002886	0.006198	0.012373
0.00041	0.002880	0.006180	0.012383
0.00041	0.002891	0.006226	0.012453
0.00042	0.002950	0.006311	0.012601
0.000419	0.002934	0.006310	0.012636
0.000426	0.002965	0.006355	0.012716
0.000421	0.002965	0.006349	0.012768
0.000428	0.002992	0.006429	0.012908
0.000433	0.003041	0.006529	0.013133
0.000436	0.003067	0.006625	0.013291
0.000441	0.003107	0.006671	0.013347
0.000431	0.003043	0.006542	0.013221

R1	R7	R15	R30
0.000426	0.002986	0.006463	0.012992
0.000419	0.002938	0.006359	0.012745
0.000418	0.002920	0.006274	0.012589
0.000419	0.002926	0.006297	0.012626
0.000418	0.002928	0.006318	0.012608
0.000417	0.002924	0.006295	0.012642
0.000423	0.002935	0.006315	0.012639
0.000424	0.002958	0.006378	0.012775
0.000417	0.002920	0.006289	0.012677
0.000415	0.002911	0.006286	0.012553
0.000415	0.002904	0.006254	0.012622
0.000403	0.002825	0.006105	0.012333
0.000406	0.002829	0.006080	0.012206
0.000407	0.002834	0.006092	0.012179
0.000405	0.002838	0.006095	0.012214
0.000403	0.002829	0.006082	0.012231
0.000408	0.002839	0.006081	0.012223
0.000406	0.002853	0.006152	0.012409
0.000408	0.002861	0.006160	0.012363
0.000407	0.002854	0.006111	0.012264
0.000408	0.002852	0.006108	0.01225
0.000411	0.002867	0.006148	0.012294
0.000411	0.002877	0.006168	0.012328
0.000406	0.002843	0.006119	0.01228
0.000409	0.002864	0.006168	0.012306
0.000411	0.002873	0.006156	0.012314
0.000411	0.002878	0.006162	0.012304
0.000411	0.002879	0.006181	0.012432
0.000406	0.002843	0.006104	0.012267
0.000405	0.002853	0.006101	0.012253
0.000403	0.002827	0.006109	0.012411
0.000407	0.002842	0.006098	0.012217
0.000408	0.002856	0.006121	0.012253
0.000407	0.002858	0.006148	0.012303
0.000409	0.002864	0.006165	0.012418
0.000407	0.002858	0.006135	0.012339
0.000408	0.002852	0.006133	0.01232
0.000407	0.002848	0.006119	0.012274
0.000408	0.002855	0.006125	0.012269
0.000405	0.002829	0.006103	0.012209
0.000406	0.002834	0.006110	0.012234
0.000404	0.002834	0.006108	0.012199
0.000408	0.002855	0.006120	0.012244
0.000408	0.002854	0.006125	0.01225
0.000408	0.002854	0.006129	0.012245
0.000409	0.002859	0.006128	0.01226
0.000409	0.002857	0.006130	0.01226
0.00041	0.002861	0.006140	0.012286
0.000417	0.002885	0.006175	0.012359
0.000412	0.002885	0.006183	0.012434
0.000415	0.002900	0.006280	0.012585
0.000431	0.002917	0.006272	0.012513
0.000419	0.002929	0.006283	0.012562
0.000408	0.002859	0.006204	0.01235
0.000406	0.002847	0.006123	0.012287
0.000399	0.002824	0.006101	0.012236

R1	R7	R15	R30
0.000404	0.002842	0.006121	0.012246
0.000406	0.002843	0.006131	0.012299
0.000404	0.002837	0.006081	0.012126
0.000404	0.002835	0.006077	0.012101
0.000405	0.002835	0.006057	0.01197
0.000405	0.002821	0.005999	0.011822
0.00038	0.002751	0.005966	0.011591