

## DECOUPLING OF SUPERSYMMETRIC PARTICLES IN THE MSSM

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### ABSTRACT

A heavy supersymmetric spectrum at the Minimal Supersymmetric Standard Model is considered and the decoupling from the low energy electroweak scale is analyzed. A formal and partial proof of decoupling of supersymmetric particles in the limit where their masses are larger than the electroweak scale is performed by integrating out all the sparticles to one loop and by evaluating the effective action for the standard electroweak gauge bosons  $W^\pm, Z$  and  $\gamma$ . The Higgs sector is not considered here. Analytical results for the two-point functions of the electroweak gauge bosons and the  $S, T$  and  $U$  parameters, to be valid in that limit, are also presented. A discussion on how the decoupling takes place in terms of both the physical sparticle masses and the non-physical mass parameters as the  $\mu$ -parameter and the soft-breaking parameters is included.

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## 1 Introduction

At present, there are indications that when the spectrum of supersymmetric (SUSY) particles at the Minimal Supersymmetric Standard Model (MSSM) is considered much heavier than the low energy electroweak scale they decouple from the low energy physics, even at the quantum level, and the resulting low energy effective theory is the Standard Model (SM) itself. However, a rigorous proof of decoupling is still lacking. On one hand there are numerical studies of observables that measure electroweak radiative corrections, like  $\Delta r$  and  $\Delta\rho^1$ , or the  $S, T$  and  $U$  parameters<sup>2</sup> as well as in the  $Z$  boson, top quark and Higgs decays<sup>3</sup>, which indicate that the one loop corrections from supersymmetric particles decrease up to negligible values in the limit of very heavy sparticle masses. Decoupling of SUSY particles is also found in some analytical studies of these and related observables<sup>1,5</sup>, as well as some computations of the effective potential for the scalar sector<sup>6</sup>, in the asymptotic limits where some of the SUSY mass parameters are considered infinitely heavy.

The question whether the Decoupling Theorem<sup>7</sup> applies or not in the case of heavy sparticles in MSSM<sup>8</sup> is not obvious at all, in our opinion. The MSSM is a gauge theory which incorporates the spontaneous symmetry breaking  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$  and chiral fermions as the SM and therefore, the direct application of the Decoupling Theorem should, in the principle, be questioned<sup>9</sup>. Examples where the Decoupling Theorem does not hold are well known. Particularly interesting are the cases of the Higgs particle and the top quark in the SM which are known not to decouple from low energy physics<sup>10,11,12</sup>.

In our opinion, a formal proof of decoupling must involve the explicit computation of the effective action by integrating out one by one all the sparticles in the MSSM to all orders in perturbation theory, and by considering the heavy sparticle masses limit. The proof will be conclusive if the remaining effective action, to be valid at energies much lower than the supersymmetric particle masses, turns out to be that of the SM with all the SUSY effects being absorbed into a redefinition of the SM parameters or else they are suppressed by inverse powers of the SUSY particle masses and vanish in the infinite masses limit.

In the present work we discuss part of the effective action which results by integrating out all the SUSY particles of the MSSM, except the Higgs sector, at the one loop level. This is a reduced version of a more complete paper to which we refer the reader for a more detailed discussion<sup>13</sup>. The integration of the Higgs sector will be considered separately in a forthcoming work<sup>14</sup>. The part of the effective action we have chosen to start with is the one for the electroweak gauge bosons,  $W^\pm$ ,  $Z$  and  $\gamma$  and, in particular, we have devoted more attention on the derivation of the two point functions with external  $W^\pm$ ,  $Z$  and  $\gamma$  gauge bosons. This will allow us to derive, in addition, the contributions from the SUSY particles to the  $S$ ,  $T$  and  $U$  parameters in the large SUSY masses limit and to conclude how the decoupling really occurs in these parameters. In order to keep our computation of the heavy SUSY particle quantum effects in a general form we have chosen to work with the masses themselves. They are the proper parameters of the large mass expansions instead of another more model dependent choices as the  $\mu$ -parameter or the soft-SUSY-breaking parameters,  $M_{\tilde{Q}}, M_{\tilde{U}}, M_{\tilde{D}}, M_{\tilde{L}}, M_{\tilde{E}}$ , and  $M_1, M_2$ <sup>a</sup>. We have considered the physically plausible situation where all the sparticle masses are large as compared to the electroweak scale but they are allowed, in principle, to be different from each other. We will explore the interesting question of to what extent the usual hypothesis of SUSY masses being generated by soft-SUSY-breaking terms and the universality of the mass parameters do or do not play a relevant role in getting decoupling. In fact, we will show in this paper, that the basic requirement of  $SU(2)_L \times U(1)_Y$  gauge invariance on the SUSY breaking terms is sufficient to obtain decoupling in the MSSM.

Finally, we have dedicated special attention and have been very careful in evaluating analytically the large SUSY masses limit of the Green functions. For this purpose, we have applied the so-called m-Theorem<sup>16</sup> which provides a rigorous technique to compute Feynman integrals with both large and small masses in the asymptotic regime of the large masses being very heavy.

The paper is organized as follows: In section 2 we present a brief discussion on how to get large mass values for all the squarks, sleptons, neutralinos and charginos at the MSSM. The third section is devoted to present the effective action for the electroweak gauge bosons  $W^\pm$ ,  $Z$  and  $\gamma$  in the MSSM that results by integrating out, in the path integral, squarks, sleptons, charginos and neutralinos to one-loop. The asymptotic results in the large SUSY masses limit for the  $\Sigma^{XY}$  and  $R^{XY}$  functions are also included and analyzed. The decoupling of heavy sparticles in the  $S$ ,  $T$  and  $U$  parameters is analyzed in the section 4. Explicit formulae for these parameters in the large SUSY masses limit as well as a discussion on these results are also presented in this section. Finally, the conclusions are summarized in section 5.

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<sup>a</sup>We use here the same notation as in<sup>8,13,15</sup>

## 2 Heavy supersymmetric spectrum at the MSSM

In this section we consider the mass eigenstates of the MSSM. Any set of particles of a given spin, baryon number, lepton number and the same  $SU(3)_c \times U(1)_{\text{em}}$  quantum numbers can mix. Therefore, in principle, there can be mixing in all the sectors of the MSSM and one must diagonalize mass matrices to obtain the mass eigenstates and the corresponding eigenvalues<sup>8,15</sup>. We consider here all the sectors, except the Higgs sector that we prefer to analyze elsewhere<sup>14</sup>.

In the present work we are interested in the Green functions with external electroweak gauge bosons and in the large mass limit of the SUSY particles, which means the situation where all the sparticle masses are much larger than the electroweak scale and the external momenta. In particular this could be the case if the sparticle masses are well above  $m_z, m_w$  and  $m_t$  but still below the few TeV upper bound that is imposed by the standard solution of the hierarchy problem. Furthermore, unless we are in a particular model, the masses of the various sparticles are, in general, different and independent. Therefore, we must take these masses to be large as compared to the external gauge boson masses and external momenta, but we must specify, in addition, how they compare to each other. More specifically, we assume here the most plausible situation where all the sparticle masses are large but close to each other; namely  $\tilde{m}_i^2, \tilde{m}_j^2 \gg M_{EW}^2, k^2$  and  $|\tilde{m}_i^2 - \tilde{m}_j^2| \ll |\tilde{m}_i^2 + \tilde{m}_j^2|$ , where  $M_{EW}$  denotes any of the electroweak masses involved ( $m_z, m_w, m_t, \dots$ ) and  $k$  denotes any external momentum. Notice that this includes the case that has been the most studied in the literature where universality of sparticle masses is assumed.

This masses hypothesis, together with the requirement that all the sparticles must be heavier than their corresponding partners, imply some constraints on the SUSY parameters<sup>13</sup>. In particular, in the squarks sector, if we ignore mixing between different generations to avoid unacceptable large flavor changing neutral currents and if we use the notation of the third family for the mass eigenstates  $\tilde{t}_1, \tilde{t}_2, \tilde{b}_1, \tilde{b}_2$  and the corresponding mass squared eigenvalues by  $\tilde{m}_{\tilde{t}_{1,2}}^2, \tilde{m}_{\tilde{b}_{1,2}}^2$ , the previous conditions imply the following constraints on the soft SUSY breaking and  $\mu$  parameters:

$$M_{\tilde{Q}}^2, M_{\tilde{U}}^2 \gg m_t^2, m_z^2, \quad |M_{\tilde{Q}}^2 - M_{\tilde{U}}^2| \ll |M_{\tilde{Q}}^2 + M_{\tilde{U}}^2|, \quad m_t^2(A_t - \mu \cot \beta)^2 < M_{\tilde{Q}}^2 M_{\tilde{U}}^2. \quad (1)$$

Here  $A_t$  is the trilinear coupling and  $\cot \beta \equiv v_1/v_2$ . The first condition implies, in turn, the limiting behaviour  $\tilde{m}_{\tilde{t}_1}^2 \rightarrow M_{\tilde{Q}}^2, \tilde{m}_{\tilde{t}_2}^2 \rightarrow M_{\tilde{U}}^2$ . The second condition means that  $M_{\tilde{Q}}$  and  $M_{\tilde{U}}$  must be close to each other and the third one means that the mixing can never be large. Similar conclusions can be obtained for the sbottoms.

In summary, in order to get large stop and sbottom masses one needs large values of the SUSY breaking masses  $M_{\tilde{Q}}, M_{\tilde{U}}$  and  $M_{\tilde{D}}$  as compared to the electroweak scale and, in order not to get a too large mixing, the trilinear couplings  $A_t, A_b$  and the  $\mu$  parameter must be constrained from above by the previous inequalities. Notice that an arbitrarily large  $\mu$  or  $A_t, A_b$  with  $M_{\tilde{Q}}, M_{\tilde{U}}, M_{\tilde{D}}$  fixed is not allowed.

Similar analysis can be done in the sleptons sector. In this sector  $\tilde{\nu}, \tilde{\tau}_1, \tilde{\tau}_2$  are the mass eigenstates and the mass squared eigenvalues are  $\tilde{m}_{\tilde{\nu}}^2, \tilde{m}_{\tilde{\tau}_1}^2, \tilde{m}_{\tilde{\tau}_2}^2$  respectively. In this case, we conclude that large squared sparticle masses, such that their sum be larger than their difference, implies that  $M_{\tilde{L}}^2$  and  $M_{\tilde{E}}^2$  are large, satisfying also  $|M_{\tilde{L}}^2 - M_{\tilde{E}}^2| \ll |M_{\tilde{L}}^2 + M_{\tilde{E}}^2|$ . Neither  $\mu$  nor  $A_\tau$  can be taken arbitrarily large with  $M_{\tilde{L}}, M_{\tilde{E}}$  fixed.

Concerning to the inos sector and by following the standard notation<sup>8,13,15</sup>, we have denoted by  $\tilde{\chi}_1^+, \tilde{\chi}_2^+$  the 4-component Dirac fermions that represent the two physical charginos and by  $\tilde{M}_{1,2}^+$  their corresponding mass eigenvalues. The 4-component Majorana fermions which represent the 4 neutralinos are denoted by  $\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0$  and  $\tilde{\chi}_4^0$  and their corresponding mass eigenvalues are  $\tilde{M}_{1,2,3,4}^0$ .

In principle, the eigenvalues in the inos sector can be either positive or negative. We choose the SUSY breaking parameters  $M_1$  and  $M_2$  to be positive and allow  $\mu$  to be either positive or negative. The physical masses,  $|\tilde{M}_{1,2}^+|$  and  $|\tilde{M}_{1,2,3,4}^0|$  are, of course, positive<sup>13</sup>. In case of negative eigenvalues we proceed following the method described in the second paper of<sup>15</sup>.

Notice that to reach the large SUSY masses limit that we are interested in, it is necessary to consider the

mass parameters in the chargino sector in the range  $M_2, \mu \gg m_w$  and therefore, to a very good approximation, the mixing is small and  $\tilde{\chi}_1^+$  will be predominantly gaugino with a mass close to  $M_2$ , whereas  $\tilde{\chi}_2^+$  will be predominantly Higgsino with a mass close to  $|\mu|$ .

Analogously, in the large SUSY masses limit important simplifications do occur in the neutralino sector<sup>13</sup>. In order to get the four neutralino masses larger than the electroweak scale it is necessary to consider the mass parameters in the range  $M_1, M_2, \mu \gg m_z$ . Therefore, to a very good approximation, the off-diagonal terms of the mass matrix in the  $(\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0)$  basis are negligible as compared to  $M_1, M_2$  and  $\mu$ . The physical mass eigenstates  $\tilde{\chi}_i^0$ , ( $i = 1, \dots, 4$ ) are predominantly  $\tilde{B}, \tilde{W}_3, (\tilde{H}_1^0 + \tilde{H}_2^0)/\sqrt{2}$  and  $(\tilde{H}_1^0 - \tilde{H}_2^0)/\sqrt{2}$ , and their corresponding masses are close to  $M_1, M_2, |\mu|$  and  $|\mu|$  respectively.

### 3 Effective action for the electroweak gauge bosons to one-loop in the large SUSY masses limit.

Our aim is to compute the effective action for the standard particles,  $\Gamma_{eff}[\phi]$ , that is defined through functional integration of all the sparticles of the MSSM. In short notation it is defined by,

$$e^{i\Gamma_{eff}[\phi]} = \int [d\tilde{\phi}] e^{i\Gamma_{\text{MSSM}}[\phi, \tilde{\phi}]}, \quad \Gamma_{\text{MSSM}}[\phi, \tilde{\phi}] \equiv \int dx \mathcal{L}_{\text{MSSM}}(\phi, \tilde{\phi}); \quad dx \equiv d^4x, \quad (2)$$

where  $\phi = l, q, A, W^\pm, Z, g, H$  are the SM particles;  $\tilde{\phi} = \tilde{l}, \tilde{q}, \tilde{A}, \tilde{W}^\pm, \tilde{Z}, \tilde{g}, \tilde{H}$  their supersymmetric partners, and  $\mathcal{L}_{\text{MSSM}}$  is the lagrangian of the MSSM<sup>13</sup>.

In this paper, we are interested in the part of the effective action that contains the two point Green functions with external gauge bosons  $\Gamma_{\mu\nu}^{XY}(k)$ ,  $X, Y = A, Z, W^\pm$ . This will allow us to study the decoupling properties of the gauge boson self-energies and from them we will deduce the corresponding analytical expressions for the well known parameters  $S, T$  and  $U$ . The computation of the effective action has been performed at the one loop level by using dimensional regularization and including the integration of all the sfermions  $\tilde{f}$ , the neutralinos  $\tilde{\chi}^0$  and the charginos  $\tilde{\chi}^\pm$ .

The effective action can be written as:

$$e^{i\Gamma_{eff}[V]} = \int [d\tilde{f}] [d\tilde{f}^*] [d\tilde{\chi}^+] [d\tilde{\chi}^0] [d\tilde{\chi}^\pm] e^{i\Gamma_{\text{MSSM}}[V, \tilde{f}, \tilde{\chi}^+, \tilde{\chi}^0]} \quad (3)$$

where  $\tilde{f} = \tilde{q}, \tilde{l}$ ;  $V = W^\pm, Z, A$  and:

$$\Gamma_{\text{MSSM}}[V, \tilde{f}, \tilde{\chi}^+, \tilde{\chi}^0] \equiv \Gamma_0[V] + \Gamma_{\tilde{f}}[V, \tilde{f}] + \Gamma_{\tilde{\chi}}[V, \tilde{\chi}] + \Gamma_{\tilde{\chi}}[\tilde{f}, \tilde{\chi}, f]. \quad (4)$$

In this formula  $\Gamma_0[V]$  is the quadratic action for gauge bosons which is taken generically in an arbitrary  $R_\xi$  gauge, and  $\Gamma_{\tilde{f}}[V, \tilde{f}]$  and  $\Gamma_{\tilde{\chi}}[V, \tilde{\chi}]$  are the actions for the sfermions and the neutralinos and charginos respectively. Notice that the last term  $\Gamma_{\tilde{\chi}}[\tilde{f}, \tilde{\chi}, f]$ , which includes the interactions among  $\tilde{f}, \tilde{\chi}$  and  $f$ , does not contribute to  $\Gamma_{eff}[V]$  to one-loop. Therefore, the formula of the effective action (3) can be factorized into three pieces:

$$e^{i\Gamma_{eff}[V]} = e^{i\Gamma_0[V]} e^{i\Gamma_{eff}^{\tilde{f}}[V]} e^{i\Gamma_{eff}^{\tilde{\chi}}[V]} \quad (5)$$

where,

$$e^{i\Gamma_{eff}^{\tilde{f}}[V]} = \int [d\tilde{f}] [d\tilde{f}^*] e^{i\Gamma_{\tilde{f}}[V, \tilde{f}]}, \quad e^{i\Gamma_{eff}^{\tilde{\chi}}[V]} = \int [d\tilde{\chi}^+] [d\tilde{\chi}^0] [d\tilde{\chi}^\pm] e^{i\Gamma_{\tilde{\chi}}[V, \tilde{\chi}]} \quad (6)$$

In order to perform the functional integration, it is convenient to write the classical action in terms of operators. We computed  $\Gamma_{eff}^{\tilde{f}}[V]$  and  $\Gamma_{eff}^{\tilde{\chi}}[V]$  separately by using the standard path integral techniques. The details of the computation can be seen in<sup>13</sup>.

If we keep just the terms that contribute to the two-point functions, we find that the effective action generated from sfermions and charginos and neutralinos integration can be written as,

$$\Gamma_{eff}^{\tilde{f}}[V] = i\text{Tr}(A_{\tilde{f}}^{(0)-1} A_{\tilde{f}}^{(2)}) - \frac{i}{2}\text{Tr}(A_{\tilde{f}}^{(0)-1} A_{\tilde{f}}^{(1)})^2 + O(V^3), \quad (7)$$

$$\Gamma_{eff}^{\tilde{\chi}}[V] = \frac{i}{2}\text{Tr}(A_+^{(o)-1} A_+^{(1)})^2 + \frac{i}{4}\text{Tr}(A_o^{(o)-1} A_o^{(1)})^2 + i\text{Tr}(A_o^{(o)-1} A_{o+}^{(1)} A_+^{(o)-1} A_{+o}^{(1)}) + O(V^3), \quad (8)$$

where the operators are,

$$\begin{aligned} A_{\tilde{f}xy}^{(0)} &\equiv (-\square - \tilde{M}_{\tilde{f}}^2)_x \delta_{xy}, & A_{0xy}^{(0)} &\equiv (i\cancel{\partial} - \tilde{M}^0)_x \delta_{xy}, & A_{+xy}^{(0)} &\equiv (i\cancel{\partial} - \tilde{M}^+)_x \delta_{xy} \\ A_{\tilde{f}xy}^{(1)} &\equiv \left[ -ie \left( \partial_\mu A^\mu \hat{Q}_f + 2 \hat{Q}_f A_\mu \partial^\mu \right) - \frac{ig}{c_w} \left( \partial_\mu Z^\mu \hat{G}_f + 2 \hat{G}_f Z_\mu \partial^\mu \right) \right. \\ &\quad \left. - \frac{ig}{\sqrt{2}} \left( \partial_\mu W^{+\mu} \Sigma_f^{tb} + 2 \Sigma_f^{tb} W_\mu^+ \partial^\mu \right) \right]_x \delta_{xy} + \text{h.c.} \\ A_{\tilde{f}xy}^{(2)} &\equiv \left( e^2 \hat{Q}_f^2 A_\mu A^\mu + \frac{2ge}{c_w} A_\mu Z^\mu \hat{Q}_f \hat{G}_f + \frac{g^2}{c_w^2} \hat{G}_f^2 Z_\mu Z^\mu + \frac{1}{2} g^2 \Sigma_f W_\mu^+ W^{\mu-} \right)_x \delta_{xy} \\ A_{0xy}^{(1)} &\equiv \left[ \frac{g}{c_w} Z_\mu \gamma^\mu (O_L'' P_L + O_R'' P_R) \right]_x \delta_{xy}, & A_{+xy}^{(1)} &\equiv \left[ \frac{g}{c_w} Z_\mu \gamma^\mu (O_L' P_L + O_R' P_R) - e A_\mu \gamma^\mu \right]_x \delta_{xy} \\ A_{0+xy}^{(1)} &\equiv [g W_\mu^- \gamma^\mu (O_L P_L + O_R P_R)]_x \delta_{xy}, & A_{+0xy}^{(1)} &\equiv [g W_\mu^+ \gamma^\mu (O_L^+ P_L + O_R^+ P_R)]_x \delta_{xy}. \end{aligned} \quad (9)$$

In all these expressions and in the following, we use the compact notation<sup>13</sup> where  $\tilde{f}$  is a four-entries column vector including sfermions of all types and the sum  $\sum_{\tilde{f}}$  is over the three generations and, in the case of squarks, it runs also over the  $N_c$  color indexes. We have introduced as well two column vectors,  $\tilde{\chi}^o$ , with components  $\tilde{\chi}_i^o$ , ( $i = 1, 2, 3, 4$ ), and  $\tilde{\chi}^+$ , which components are  $\tilde{\chi}_i^+$ , ( $i = 1, 2$ ). The corresponding mass matrices are:

$$\tilde{M}_{\tilde{f}}^2 = \text{diag}(\tilde{m}_{t_1}^2, \tilde{m}_{t_2}^2, \tilde{m}_{b_1}^2, \tilde{m}_{b_2}^2) \quad \text{if } \tilde{f} = \tilde{q}; \quad \tilde{M}_{\tilde{f}}^2 = \text{diag}(\tilde{m}_{\nu}^2, 0, \tilde{m}_{\tau_1}^2, \tilde{m}_{\tau_2}^2) \quad \text{if } \tilde{f} = \tilde{l}, \quad (10)$$

$$\tilde{M}^+ = \text{diag}(\tilde{M}_1^+, \tilde{M}_2^+), \quad \tilde{M}^0 = \text{diag}(\tilde{M}_1^0, \tilde{M}_2^0, \tilde{M}_3^0, \tilde{M}_4^0) \quad (11)$$

The coupling matrices  $\hat{Q}_f, \hat{G}_f, \Sigma_f^{tb}, \Sigma_f$  and  $O_{L,R}, O'_{L,R}, O''_{L,R}$  can be found in<sup>13</sup>.

Notice that, diagrammatically, the first and second terms in eq. (7) give the two types of one-loop contributions with all kind of sfermions in the loop, the first term in eq. (8) gives the one-loop contributions with charginos in the loop, the second term is the corresponding contribution with neutralinos in the loop and the last one gives the mixed one-loop contributions with both charginos and neutralinos in the loop.

In order to get the explicit expressions for the two-point functions one must work out the traces in the above formulae. Basically one must substitute all the operators, express the one-loop integrals in momentum space of D dimensions, compute all the appearing Dirac traces and Fourier transform the result back to the position space. The traces also involve to perform the sum in the corresponding matrix indexes, the sum over the various types of sfermions and the sum in color indexes in the case of squarks. We have done this computation, in addition, by diagrammatical methods and we have found the same results. Notice that they are exact to one-loop. The result for the effective action is<sup>13</sup>:

$$\begin{aligned} \Gamma_{eff}[V] &= \frac{1}{2} \int dx dy A_x^\mu \Gamma_{\mu\nu}^{AA}(x, y) A_y^\nu + \frac{1}{2} \int dx dy Z_x^\mu \Gamma_{\mu\nu}^{ZZ}(x, y) Z_y^\nu + \frac{1}{2} \int dx dy A_x^\mu \Gamma_{\mu\nu}^{AZ}(x, y) Z_y^\nu \\ &\quad + (A \leftrightarrow Z) + \frac{1}{2} \int dx dy W_x^{+\mu} \Gamma_{\mu\nu}^{WW}(x, y) W_y^{-\nu} + (+ \leftrightarrow -) + O(V^3) \end{aligned} \quad (12)$$

where  $\Gamma_{\mu\nu}(x, y)$  are the two-point functions for the electroweak gauge bosons in position space.

The results for these two-point functions in momentum space are as follows:

$$\begin{aligned}\Gamma_{\mu\nu}^{AA}(k) &= \Gamma_{0\mu\nu}^{AA}(k) + ie^2 \sum_{\tilde{f}} \left\{ 2 \sum_a I_0(\tilde{m}_{f_a}^2) (\hat{Q}_f^2)_{aa} g_{\mu\nu} - \sum_{ab} (\hat{Q}_f)_{ab} (\hat{Q}_f)_{ba} I_{f_{\mu\nu}}^{ab}(k, \tilde{m}_{f_a}, \tilde{m}_{f_b}) \right\} \\ &+ 2ie^2 \sum_{i=1}^2 \left\{ T_{\mu\nu}^{ii}(k, \tilde{M}_i^+, \tilde{M}_i^+) + 2\hat{I}^{ii}(k, \tilde{M}_i^+, \tilde{M}_i^+) g_{\mu\nu} \right\}\end{aligned}\quad (13)$$

$$\begin{aligned}\Gamma_{\mu\nu}^{ZZ}(k) &= \Gamma_{0\mu\nu}^{ZZ}(k) + i \frac{g^2}{c_W^2} \sum_{\tilde{f}} \left\{ 2 \sum_a I_0(\tilde{m}_{f_a}^2) (\hat{G}_f^2)_{aa} g_{\mu\nu} - \sum_{ab} (\hat{G}_f)_{ab} (\hat{G}_f)_{ba} I_{f_{\mu\nu}}^{ab}(k, \tilde{m}_{f_a}, \tilde{m}_{f_b}) \right\} \\ &+ \frac{i}{2} \frac{g^2}{c_W^2} \sum_{i,j=1}^4 \left\{ (O_L^{ij} O_L^{ji} + O_R^{ij} O_R^{ji}) T_{\mu\nu}^{ij}(k, \tilde{M}_i^0, \tilde{M}_j^0) + 2(O_L^{ij} O_R^{ji} + O_R^{ij} O_L^{ji}) \hat{I}^{ij}(k, \tilde{M}_i^0, \tilde{M}_j^0) g_{\mu\nu} \right\} \\ &+ i \frac{g^2}{c_W^2} \sum_{i,j=1}^2 \left\{ (O_L^{ij} O_L^{ji} + O_R^{ij} O_R^{ji}) T_{\mu\nu}^{ij}(k, \tilde{M}_i^+, \tilde{M}_j^+) + 2(O_L^{ij} O_R^{ji} + O_R^{ij} O_L^{ji}) \hat{I}^{ij}(k, \tilde{M}_i^+, \tilde{M}_j^+) g_{\mu\nu} \right\}\end{aligned}\quad (14)$$

$$\begin{aligned}\Gamma_{\mu\nu}^{AZ}(k) &= \Gamma_{\mu\nu}^{ZA}(k) = \frac{ige}{c_W} \sum_{\tilde{f}} \left\{ 2 \sum_a I_0(\tilde{m}_{f_a}^2) (\hat{Q}_f \hat{G}_f)_{aa} g_{\mu\nu} - \sum_{ab} (\hat{Q}_f)_{ab} (\hat{G}_f)_{ba} I_{f_{\mu\nu}}^{ab}(k, \tilde{m}_{f_a}, \tilde{m}_{f_b}) \right\} \\ &- \frac{ige}{c_W} \sum_{i=1}^2 (O_L^{ii} + O_R^{ii}) \left( T_{\mu\nu}^{ii}(k, \tilde{M}_i^+, \tilde{M}_i^+) + 2\hat{I}^{ii}(k, \tilde{M}_i^+, \tilde{M}_i^+) g_{\mu\nu} \right)\end{aligned}\quad (15)$$

$$\begin{aligned}\Gamma_{\mu\nu}^{WW}(k) &= \Gamma_{0\mu\nu}^{WW}(k) + \frac{ig^2}{2} \sum_{\tilde{f}} \left\{ \sum_a (\Sigma_f)_{aa} I_0(\tilde{m}_{f_a}^2) g_{\mu\nu} - \sum_{a,b} (\Sigma_f^{tb})_{ab} (\Sigma_f^{tb})_{ab} I_{f_{\mu\nu}}^{ab}(k, \tilde{m}_{f_a}, \tilde{m}_{f_b}) \right\} \\ &+ ig^2 \sum_{i=1}^4 \sum_{j=1}^2 \left\{ (O_L^{ij} O_L^{ji} + O_R^{ij} O_R^{ji}) T_{\mu\nu}^{ij}(k, \tilde{M}_i^0, \tilde{M}_j^+) \right. \\ &\left. + 2(O_L^{ij} O_R^{ji} + O_R^{ij} O_L^{ji}) \hat{I}^{ij}(k, \tilde{M}_i^0, \tilde{M}_j^+) g_{\mu\nu} \right\}\end{aligned}\quad (16)$$

Here the indexes  $a$  and  $b$  run from one to four, corresponding to the four entries of the column vector  $\tilde{f}$ . The indexes  $i, j$  vary as  $i, j = 1, 2, 3, 4$  if they refer to neutralinos and as  $i, j = 1, 2$  if they refer to charginos.  $\Gamma_{0\mu\nu}^{VV}$  ( $V = Z, W$ ) and  $\Gamma_{0\mu\nu}^{AA}$  are the two-point functions at tree level, which are defined by:

$$\Gamma_{0\mu\nu}^{VV}(k) = (M_V - k^2) g_{\mu\nu} + \left(1 - \frac{1}{\xi_V}\right) k_\mu k_\nu; \quad \Gamma_{0\mu\nu}^{AA} = -k^2 g_{\mu\nu} + \left(1 - \frac{1}{\xi_A}\right) k_\mu k_\nu, \quad (17)$$

and the one-loop integrals  $I_0(\tilde{m}_{f_a}^2)$ ,  $I_{f_{\mu\nu}}^{ab}(k, \tilde{m}_{f_a}, \tilde{m}_{f_b})$ ,  $T_{\mu\nu}^{ij}(k, \tilde{M}_i, \tilde{M}_j)$ ,  $\hat{I}^{ij}(k, \tilde{M}_i, \tilde{M}_j)$  are defined in dimensional regularization by,

$$I_0(\tilde{m}_{f_a}^2) = \int d\hat{q} \frac{1}{[q^2 - \tilde{m}_{f_a}^2]}; \quad I_{f_{\mu\nu}}^{ab}(k, \tilde{m}_{f_a}, \tilde{m}_{f_b}) = \int d\hat{q} \frac{(2q+k)_\mu (2q+k)_\nu}{[(k+q)^2 - \tilde{m}_{f_a}^2] [q^2 - \tilde{m}_{f_b}^2]} \quad (18)$$

$$T_{\mu\nu}^{ij}(k, \tilde{M}_i, \tilde{M}_j) = \int d\hat{q} \frac{(4q_\mu q_\nu - 2g_{\mu\nu} g^{\alpha\beta} q_\alpha q_\beta + 2(q_\mu k_\nu + q_\nu k_\mu) - 2g_{\mu\nu} g^{\alpha\beta} q_\alpha k_\beta)}{[(k+q)^2 - \tilde{m}_{f_a}^2] [q^2 - \tilde{m}_{f_b}^2]} \quad (19)$$

$$\hat{I}^{ij}(k, \tilde{M}_i, \tilde{M}_j) = \int d\hat{q} \frac{\tilde{M}_i \tilde{M}_j}{[q^2 - \tilde{M}_i^2] [(k+q)^2 - \tilde{M}_j^2]} \quad (20)$$

Since we are interested in the large mass limit of the SUSY particles we need to have at hand not just the exact results of the above mentioned integrals but their asymptotic expressions to be valid in that limit. We have analyzed the integrals by means of the so-called m-Theorem<sup>16</sup>. This theorem provides a powerful technique to study the asymptotic behaviour of Feynman integrals in the limit where some of the masses are large. Notice that this is not trivial since some of these integrals are divergent and the interchange of the integral with the limit is not allowed. Thus, one should first compute the integrals in dimensional regularization and at the end take the large mass limit. Instead of this direct way it is also possible to proceed as follows: First, in order to decrease the ultraviolet divergent degree, one rearranges the integrand through algebraic manipulations up to separate the Feynman integral into a divergent part, which can be evaluated exactly using the standard techniques of dimensional regularization, and a convergent part that satisfies the requirements demanded by the m-Theorem and therefore, goes to zero in the infinite mass limit. By means of this procedure the correct asymptotic behaviour of the integrals is guaranteed. Some examples of the computation of the Feynman integrals by means of the m-Theorem as well as details of this theorem are given in<sup>13</sup>. The results for the above one loop integrals in the large masses limit are as follows:

$$\begin{aligned}
I_0(\tilde{m}_{f_a}^2) &= \frac{i}{16\pi^2} \left( \Delta_\epsilon + 1 - \log \frac{\tilde{m}_{f_a}^2}{\mu_o^2} \right) \tilde{m}_{f_a}^2 \\
I_f^{ab}(k, \tilde{m}_{f_a}, \tilde{m}_{f_b}) &= \frac{i}{16\pi^2} \left\{ (\tilde{m}_{f_a}^2 + \tilde{m}_{f_b}^2) \left( \Delta_\epsilon + 1 - \log \frac{(\tilde{m}_{f_a}^2 + \tilde{m}_{f_b}^2)}{2\mu_o^2} \right) g_{\mu\nu} - \frac{1}{3} k^2 \left( \Delta_\epsilon - \log \frac{(\tilde{m}_{f_a}^2 + \tilde{m}_{f_b}^2)}{2\mu_o^2} \right) g_{\mu\nu} \right. \\
&\quad \left. + \frac{1}{3} k_\mu k_\nu \left( \Delta_\epsilon - \log \frac{(\tilde{m}_{f_a}^2 + \tilde{m}_{f_b}^2)}{2\mu_o^2} \right) \right\} \\
T^{ij}(k, \tilde{M}_i, \tilde{M}_j) &= \frac{i}{16\pi^2} \left\{ -(\tilde{M}_i^2 + \tilde{M}_j^2) \left( \Delta_\epsilon - \log \frac{(\tilde{M}_i^2 + \tilde{M}_j^2)}{2\mu_o^2} \right) + \frac{2}{3} k^2 \left( \Delta_\epsilon - \frac{1}{2} - \log \frac{(\tilde{M}_i^2 + \tilde{M}_j^2)}{2\mu_o^2} \right) g_{\mu\nu} \right. \\
&\quad \left. - \frac{2}{3} k_\mu k_\nu \left( \Delta_\epsilon - \log \frac{(\tilde{M}_i^2 + \tilde{M}_j^2)}{2\mu_o^2} \right) \right\}, \\
\hat{I}^{ij}(k, \tilde{M}_i, \tilde{M}_j) &= \frac{i}{16\pi^2} \left\{ \frac{1}{2} (\tilde{M}_i^2 + \tilde{M}_j^2) \left( \Delta_\epsilon - \log \frac{(\tilde{M}_i^2 + \tilde{M}_j^2)}{2\mu_o^2} \right) + \frac{1}{6} k^2 \right. \\
&\quad \left. - \frac{1}{2} (\tilde{M}_i - \tilde{M}_j)^2 \left( \Delta_\epsilon - \log \frac{(\tilde{M}_i^2 + \tilde{M}_j^2)}{2\mu_o^2} \right) \right\}. \tag{21}
\end{aligned}$$

where  $\mu_o$  is the usual mass scale of dimensional regularization and,

$$\Delta_\epsilon = \frac{2}{\epsilon} - \gamma_\epsilon + \log(4\pi) \quad , \quad \epsilon = 4 - D. \tag{22}$$

Finally, we define the self-energies  $\Sigma^{XY}(k)$  and the  $R^{XY}(k)$  functions, from the two-point functions as usual,

$$\Gamma_{\mu\nu}^{XY}(k) = \Gamma_{0\mu\nu}^{XY}(k) + \Sigma^{XY}(k) g_{\mu\nu} + R^{XY}(k) k_\mu k_\nu. \tag{23}$$

The asymptotic expressions for the  $\Sigma^{XY}$  and  $R^{XY}$  functions, in the large sparticle masses limit and for each sector, can be obtained from our results of eqs.(13-16) and by using the formulae of eqs.(21). We find the following results:

### 3.1 Squarks sector:

For  $\tilde{m}_{\tilde{t}_1}^2, \tilde{m}_{\tilde{t}_2}^2, \tilde{m}_{\tilde{b}_1}^2, \tilde{m}_{\tilde{b}_2}^2 \gg k^2$ ,  $|\tilde{m}_{\tilde{t}_1}^2 - \tilde{m}_{\tilde{t}_2}^2| \ll |\tilde{m}_{\tilde{t}_1}^2 + \tilde{m}_{\tilde{t}_2}^2|$ ;  $|\tilde{m}_{\tilde{b}_1}^2 - \tilde{m}_{\tilde{b}_2}^2| \ll |\tilde{m}_{\tilde{b}_1}^2 + \tilde{m}_{\tilde{b}_2}^2|$ , and

$|\tilde{m}_{\tilde{t}_i}^2 - \tilde{m}_{\tilde{b}_j}^2| \ll |\tilde{m}_{\tilde{t}_i}^2 + \tilde{m}_{\tilde{b}_j}^2|$  ( $i, j = 1, 2$ ), we get:

$$\Sigma_{\tilde{q}}^{AA}(k) = -N_c \frac{e^2}{16\pi^2} k^2 \frac{1}{3} \sum_{\tilde{q}} \left\{ \frac{10}{9} \Delta_\epsilon - \frac{4}{9} \left( \log \frac{\tilde{m}_{\tilde{t}_1}^2}{\mu_o^2} + \log \frac{\tilde{m}_{\tilde{t}_2}^2}{\mu_o^2} \right) - \frac{1}{9} \left( \log \frac{\tilde{m}_{\tilde{b}_1}^2}{\mu_o^2} + \log \frac{\tilde{m}_{\tilde{b}_2}^2}{\mu_o^2} \right) \right\}, \quad (24)$$

$$\begin{aligned} \Sigma_{\tilde{q}}^{AZ}(k) &= -N_c \frac{e^2}{16\pi^2} k^2 \frac{1}{3s_w c_w} \sum_{\tilde{q}} \left\{ \left( \frac{1}{2} - \frac{10}{9} s_w^2 \right) \Delta_\epsilon - \frac{2}{3} \left( \frac{1}{2} c_t^2 - \frac{2}{3} s_w^2 \right) \log \frac{\tilde{m}_{\tilde{t}_1}^2}{\mu_o^2} \right. \\ &\quad \left. - \frac{2}{3} \left( \frac{1}{2} s_t^2 - \frac{2}{3} s_w^2 \right) \log \frac{\tilde{m}_{\tilde{t}_2}^2}{\mu_o^2} + \frac{1}{3} \left( -\frac{1}{2} c_b^2 + \frac{1}{3} s_w^2 \right) \log \frac{\tilde{m}_{\tilde{b}_1}^2}{\mu_o^2} + \frac{1}{3} \left( -\frac{1}{2} s_b^2 + \frac{1}{3} s_w^2 \right) \log \frac{\tilde{m}_{\tilde{b}_2}^2}{\mu_o^2} \right\}, \end{aligned} \quad (25)$$

$$\begin{aligned} \Sigma_{\tilde{q}}^{ZZ}(k) &= -N_c \frac{e^2}{16\pi^2} \frac{1}{s_w^2 c_w^2} \sum_{\tilde{q}} \left\{ \left[ -\frac{1}{2} c_t^2 s_t^2 h(\tilde{m}_{\tilde{t}_1}^2, \tilde{m}_{\tilde{t}_2}^2) - \frac{1}{2} c_b^2 s_b^2 h(\tilde{m}_{\tilde{b}_1}^2, \tilde{m}_{\tilde{b}_2}^2) \right] \right. \\ &\quad + \frac{1}{3} k^2 \left[ \left( \frac{c_t^2}{2} - \frac{2s_w^2}{3} \right)^2 \left( \Delta_\epsilon - \log \frac{\tilde{m}_{\tilde{t}_1}^2}{\mu_o^2} \right) + \left( \frac{s_t^2}{2} - \frac{2s_w^2}{3} \right)^2 \left( \Delta_\epsilon - \log \frac{\tilde{m}_{\tilde{t}_2}^2}{\mu_o^2} \right) \right. \\ &\quad + \left( -\frac{c_b^2}{2} + \frac{s_w^2}{3} \right)^2 \left( \Delta_\epsilon - \log \frac{\tilde{m}_{\tilde{b}_1}^2}{\mu_o^2} \right) + \left( -\frac{s_b^2}{2} + \frac{s_w^2}{3} \right)^2 \left( \Delta_\epsilon - \log \frac{\tilde{m}_{\tilde{b}_2}^2}{\mu_o^2} \right) \\ &\quad \left. \left. + \frac{1}{2} s_t^2 c_t^2 \left( \Delta_\epsilon - \log \frac{\tilde{m}_{\tilde{t}_1}^2 + \tilde{m}_{\tilde{t}_2}^2}{2\mu_o^2} \right) + \frac{1}{2} s_b^2 c_b^2 \left( \Delta_\epsilon - \log \frac{\tilde{m}_{\tilde{b}_1}^2 + \tilde{m}_{\tilde{b}_2}^2}{2\mu_o^2} \right) \right] \right\}, \end{aligned} \quad (26)$$

$$\begin{aligned} \Sigma_{\tilde{q}}^{WW}(k) &= -N_c \frac{e^2}{16\pi^2} \frac{1}{s_w^2} \sum_{\tilde{q}} \left\{ \left[ -\frac{1}{2} c_t^2 c_b^2 h(\tilde{m}_{\tilde{t}_1}^2, \tilde{m}_{\tilde{b}_1}^2) - \frac{1}{2} c_t^2 s_b^2 h(\tilde{m}_{\tilde{t}_1}^2, \tilde{m}_{\tilde{b}_2}^2) \right. \right. \\ &\quad \left. - \frac{1}{2} s_t^2 c_b^2 h(\tilde{m}_{\tilde{t}_2}^2, \tilde{m}_{\tilde{b}_1}^2) - \frac{1}{2} s_t^2 s_b^2 h(\tilde{m}_{\tilde{t}_2}^2, \tilde{m}_{\tilde{b}_2}^2) \right] + \frac{1}{6} k^2 \left[ \Delta_\epsilon - c_t^2 c_b^2 \log \frac{\tilde{m}_{\tilde{t}_1}^2 + \tilde{m}_{\tilde{b}_1}^2}{2\mu_o^2} \right. \\ &\quad \left. \left. - c_t^2 s_b^2 \log \frac{\tilde{m}_{\tilde{t}_1}^2 + \tilde{m}_{\tilde{b}_2}^2}{2\mu_o^2} - s_t^2 c_b^2 \log \frac{\tilde{m}_{\tilde{t}_2}^2 + \tilde{m}_{\tilde{b}_1}^2}{2\mu_o^2} - s_t^2 s_b^2 \log \frac{\tilde{m}_{\tilde{t}_2}^2 + \tilde{m}_{\tilde{b}_2}^2}{2\mu_o^2} \right] \right\}, \end{aligned} \quad (27)$$

where  $s_w^2 = \sin^2 \theta_w$  and  $c_f = \cos \theta_f, s_f = \sin \theta_f$ , with  $\theta_f$  being the mixing angle in the  $f$ -sector.

The function  $h(m_1^2, m_2^2)$  has been defined in<sup>13</sup> and its asymptotic behaviour for large masses  $m_1^2, m_2^2$  and  $|m_1^2 - m_2^2| \ll |m_1^2 + m_2^2|$  is given by:

$$h(m_1^2, m_2^2) \longrightarrow \frac{m_1^2 - m_2^2}{2} \left[ \frac{(m_1^2 - m_2^2)}{(m_1^2 + m_2^2)} + O\left( \frac{m_1^2 - m_2^2}{m_1^2 + m_2^2} \right)^2 \right] \quad (28)$$

For the  $R^{XY}(k)$  functions we find expressions which are similar to the part proportional to  $k^2$  in  $\Sigma^{XY}(k)$  and with opposite sign. We omit to write the explicit formulae here for brevity. They can, generically, be written as:

$$R^{XY}(k) = - [\text{term in } k^2 \text{ of } \Sigma^{XY}(k)] / k^2 \quad (29)$$

As can be seen from eqs.(24-27), the asymptotic results in the large SUSY masses limit are of the generic form:  $\Sigma^{XY}(k) = \Sigma_{(0)}^{XY} + \Sigma_{(1)}^{XY} k^2$  and  $R^{XY}(k) = R_{(0)}^{XY}$ , where  $\Sigma_{(0)}^{XY}, \Sigma_{(1)}^{XY}$  and  $R_{(0)}^{XY}$  are functions of the large SUSY



masses but are  $k$  independent. As can be easily shown, it implies that all the remaining dependence on the SUSY masses can be absorbed into a redefinition of the SM relevant parameters,  $m_w, m_z$  and  $e$  and the gauge bosons wave functions. Therefore, the decoupling of squarks in the two point functions does indeed occur.

Similar results have been obtained in the sleptons sector.

### 3.2 Charginos and neutralinos sector:

For  $\tilde{M}_1^{+2}, \tilde{M}_2^{+2} \gg k^2$ ,  $|\tilde{M}_1^{+2} - \tilde{M}_2^{+2}| \ll |\tilde{M}_1^{+2} + \tilde{M}_2^{+2}|$ ;  $|\tilde{M}_i^{o2} - \tilde{M}_j^{o2}| \ll |\tilde{M}_i^{o2} + \tilde{M}_j^{o2}|$  ( $i, j = 1, 2, 3, 4$ ), and  $|\tilde{M}_i^{o2} - \tilde{M}_j^{+2}| \ll |\tilde{M}_i^{o2} + \tilde{M}_j^{+2}|$ , ( $i = 1, 2, 3, 4$ ); ( $j = 1, 2$ ), we find:

$$\Sigma_{\tilde{\chi}}^{AA}(k) = -\frac{e^2}{16\pi^2} \frac{4}{3} k^2 \left( 2\Delta_\epsilon - \log \frac{\tilde{M}_1^{+2}}{\mu_o^2} - \log \frac{\tilde{M}_2^{+2}}{\mu_o^2} \right) \quad (30)$$

$$\Sigma_{\tilde{\chi}}^{AZ}(k) = \frac{e^2}{16\pi^2} \frac{1}{s_w c_w} \frac{4}{3} k^2 \left\{ (s_w^2 - 1) \left( \Delta_\epsilon - \log \frac{\tilde{M}_1^{+2}}{\mu_o^2} \right) + \left( s_w^2 - \frac{1}{2} \right) \left( \Delta_\epsilon - \log \frac{\tilde{M}_2^{+2}}{\mu_o^2} \right) \right\}, \quad (31)$$

$$\begin{aligned} \Sigma_{\tilde{\chi}}^{ZZ}(k) &= -\frac{e^2}{16\pi^2} \frac{1}{s_w^2 c_w^2} \left\{ -\frac{1}{2} (\tilde{M}_3^o - \tilde{M}_4^o)^2 \left( \Delta_\epsilon - \log \frac{\tilde{M}_3^{o2} + \tilde{M}_4^{o2}}{2\mu_o^2} \right) \right. \\ &+ \frac{4}{3} k^2 \left[ (s_w^2 - 1)^2 \left( \Delta_\epsilon - \log \frac{\tilde{M}_1^{+2}}{\mu_o^2} \right) + \left( s_w^2 - \frac{1}{2} \right)^2 \left( \Delta_\epsilon - \log \frac{\tilde{M}_2^{+2}}{\mu_o^2} \right) \right. \\ &\left. \left. + \frac{1}{4} \left( \Delta_\epsilon - \log \frac{\tilde{M}_3^{o2} + \tilde{M}_4^{o2}}{2\mu_o^2} \right) \right] \right\}, \quad (32) \end{aligned}$$

$$\begin{aligned} \Sigma_{\tilde{\chi}}^{WW}(k) &= -\frac{e^2}{16\pi^2} \frac{1}{s_w^2} \left\{ -2(\tilde{M}_1^+ - \tilde{M}_2^o)^2 \left( \Delta_\epsilon - \log \frac{\tilde{M}_1^{+2} + \tilde{M}_2^{o2}}{2\mu_o^2} \right) \right. \\ &- \frac{1}{2} (\tilde{M}_2^+ - \tilde{M}_3^o)^2 \left( \Delta_\epsilon - \log \frac{\tilde{M}_2^{+2} + \tilde{M}_3^{o2}}{2\mu_o^2} \right) - \frac{1}{2} (\tilde{M}_2^+ - \tilde{M}_4^o)^2 \left( \Delta_\epsilon - \log \frac{\tilde{M}_2^{+2} + \tilde{M}_4^{o2}}{2\mu_o^2} \right) \\ &\left. + k^2 \left[ 2\Delta_\epsilon - \frac{4}{3} \log \frac{\tilde{M}_1^{+2} + \tilde{M}_2^{o2}}{2\mu_o^2} - \frac{1}{3} \log \frac{\tilde{M}_2^{+2} + \tilde{M}_3^{o2}}{2\mu_o^2} - \frac{1}{3} \log \frac{\tilde{M}_2^{+2} + \tilde{M}_4^{o2}}{2\mu_o^2} \right] \right\}, \quad (33) \end{aligned}$$

Similarly to the squarks sector, the results for the  $R^{XY}(k)$  functions, can be written generically as in eq. (29).

If we consider the large ino masses limit which, as we have said in the above section, implies  $\tilde{M}_1^{+2} \rightarrow M_2^2$ ,  $\tilde{M}_2^{+2} \rightarrow \mu^2$ ,  $\tilde{M}_1^{o2} \rightarrow M_1^2$ ,  $\tilde{M}_2^{o2} \rightarrow M_2^2$  and  $\tilde{M}_3^{o2} = \tilde{M}_4^{o2} \rightarrow \mu^2$ , it is easy to see that the remaining dependence on the SUSY masses is logarithmic. The  $\Sigma^{XY}(k)$  functions are proportional to  $k^2$ , and the  $R^{XY}(k)$  functions are  $k$  independent. Therefore, similarly to the squarks sector, the decoupling of inos takes place.

In summary, from our results it is clear that there is indeed decoupling in the two point electroweak gauge boson functions: All SUSY effects can be absorbed into redefinitions of  $m_z, m_w, e$  and the wave functions of the gauge bosons  $W^\pm, Z, A$ , or else they are suppressed by inverse powers of the heavy SUSY particles masses.

In addition, some comments can be done. First, these asymptotic expressions are completely general and depend just on the physical masses of the SUSY particles and on the generic coefficients  $c_q, s_q, c_l, s_l, O_{L,R}^{ji}, O_{L,R}^{ji}$ . Notice that they do not depend on the particular mechanism that generates the SUSY masses.

Second, it is worth to mention that in order to get the above asymptotic expressions not all the SUSY masses need to be compared with each other, but just the ones appearing in the same one-loop diagram. Thus, for instance, the self-energies  $\Sigma^{AA}$  and  $\Sigma^{AZ}$ , where no mixed diagrams with different sfermions contribute, do not need of any reference on the relative size of the sfermion masses.  $\Sigma^{ZZ}$  and  $\Sigma^{WW}$ , on the contrary, do require this comparison. In the case of  $\Sigma^{ZZ}$  one needs to compare squarks of the same charge, sleptons of the same charge, charginos of the same charge and neutralinos among them. No comparison among sfermions of different generations is required since we have not considered intergenerational mixing in this work. In the case of  $\Sigma^{WW}$  one needs to compare, in each generation, the squarks of different charge, the sleptons of different charge and the neutralinos with the charginos. The realistic and more interesting situation will be when all the sparticles masses must be compared at the same time and, obviously, the final result will depend on the kind of SUSY hierarchy masses that had been previously established. This will happen in the observables as  $S, T$  and  $U$  where all the four self-energies do contribute.

#### 4 Decoupling of sparticles in $S, T$ and $U$

The radiative corrections from SUSY particles to the observables  $S, T$  and  $U$  have been analyzed exhaustively in the literature<sup>1,2</sup>, but neither their complete analytical expressions in the large sparticle masses limit nor a general and systematic study of sparticles decoupling have been provided so far. We have obtained the results to one loop for the analytical expressions of  $S, T$  and  $U$  in a complete general form and analyzed under which particular conditions the sparticles decoupling takes place, discussing how and why does it occur in the very special case of the MSSM with soft SUSY breaking terms<sup>13</sup>.

The definition that we use for  $S, T$  and  $U$  are the usual ones<sup>17</sup>. The contribution to  $S, T$  and  $U$  are known to be finite and well defined separately for each sparticle sector, so that we can analyze them separately as well. As we have already said we consider in this paper all the sparticle contributions except that of the Higgs sector.

The results for  $S, T$  and  $U$  can be obtained easily by using the corresponding expressions for the self-energies given eqs. (24-33). Notice that, for each case, we must consider the corresponding conditions on the masses above mentioned. Although the three parameters,  $S, T$  and  $U$  do not require the same set of conditions on the sparticle masses, the physical and realistic situation corresponds to have fixed all the SUSY spectra at once, and therefore all these conditions must hold together.

##### 4.1 Squarks sector:

By considering the conditions given at the beginning of section 3.1 together, is equivalent to say that all the squarks of the same generation have masses of similar large size. Interestingly, if we look just at the  $S_{\bar{q}}$  parameter, there is apparently no decoupling since the dominant contribution goes as<sup>13</sup>:

$$S_{\bar{q}} \rightarrow - \sum_{\bar{q}} \frac{N_c}{36\pi} \log \frac{\tilde{m}_{t_1}^2}{\tilde{m}_{b_1}^2}, \quad (\tilde{m}_{q_i}^2 \gg k^2) \quad (34)$$

which under the corresponding conditions  $|\tilde{m}_{t_1}^2 - \tilde{m}_{t_2}^2| \ll |\tilde{m}_{t_1}^2 + \tilde{m}_{t_2}^2|$  and  $|\tilde{m}_{b_1}^2 - \tilde{m}_{b_2}^2| \ll |\tilde{m}_{b_1}^2 + \tilde{m}_{b_2}^2|$  does not vanish in the infinite  $\tilde{m}_{t_1}$  and  $\tilde{m}_{b_1}$  limit. However, when the three parameters  $S_{\bar{q}}, T_{\bar{q}}$  and  $U_{\bar{q}}$  are analyzed together and the extra condition  $|\tilde{m}_{t_1}^2 - \tilde{m}_{b_1}^2| \ll |\tilde{m}_{t_1}^2 + \tilde{m}_{b_1}^2|$  is included, then the above dominant term in  $S_{\bar{q}}$  also vanishes in the infinite squark masses limit as it was expected.

In order to show the decoupling explicitly one must make an expansion of  $S_{\bar{q}}, T_{\bar{q}}$  and  $U_{\bar{q}}$  in powers of the

proper expansion parameters, which in the (third generation) squarks sector are:

$$\frac{\tilde{m}_{t_1}^2 - \tilde{m}_{b_1}^2}{\tilde{m}_{t_1}^2 + \tilde{m}_{b_1}^2}, \quad \frac{\tilde{m}_{b_1}^2 - \tilde{m}_{b_2}^2}{\tilde{m}_{b_1}^2 + \tilde{m}_{b_2}^2}, \quad \frac{\tilde{m}_{t_i}^2 - \tilde{m}_{b_j}^2}{\tilde{m}_{t_i}^2 + \tilde{m}_{b_j}^2}, \quad (i, j = 1, 2).$$

In terms of these parameters we get,

$$\begin{aligned} S_{\tilde{q}} \rightarrow & - \sum_{\tilde{q}} \frac{N_c}{18\pi} \left\{ \left( \frac{\tilde{m}_{t_1}^2 - \tilde{m}_{b_1}^2}{\tilde{m}_{t_1}^2 + \tilde{m}_{b_1}^2} \right) + s_b^2 \left( \frac{\tilde{m}_{b_1}^2 - \tilde{m}_{b_2}^2}{\tilde{m}_{b_1}^2 + \tilde{m}_{b_2}^2} \right) - s_t^2 \left( \frac{\tilde{m}_{t_1}^2 - \tilde{m}_{t_2}^2}{\tilde{m}_{t_1}^2 + \tilde{m}_{t_2}^2} \right) \right. \\ & \left. - 3c_t^2 s_t^2 \left( \frac{\tilde{m}_{t_1}^2 - \tilde{m}_{t_2}^2}{\tilde{m}_{t_1}^2 + \tilde{m}_{t_2}^2} \right)^2 - 3c_b^2 s_b^2 \left( \frac{\tilde{m}_{b_1}^2 - \tilde{m}_{b_2}^2}{\tilde{m}_{b_1}^2 + \tilde{m}_{b_2}^2} \right)^2 \right\} + O \left( \frac{\tilde{m}_i^2 - \tilde{m}_j^2}{\tilde{m}_i^2 + \tilde{m}_j^2} \right)^3, \end{aligned} \quad (35)$$

$$\begin{aligned} T_{\tilde{q}} \rightarrow & \sum_{\tilde{q}} \frac{N_c}{16\pi} \frac{1}{s_W^2 m_W^2} \left\{ c_t^2 c_b^2 (\tilde{m}_{t_1}^2 - \tilde{m}_{b_1}^2) \left( \frac{\tilde{m}_{t_1}^2 - \tilde{m}_{b_1}^2}{\tilde{m}_{t_1}^2 + \tilde{m}_{b_1}^2} \right) + c_t^2 s_b^2 (\tilde{m}_{t_1}^2 - \tilde{m}_{b_2}^2) \left( \frac{\tilde{m}_{t_1}^2 - \tilde{m}_{b_2}^2}{\tilde{m}_{t_1}^2 + \tilde{m}_{b_2}^2} \right) \right. \\ & - c_t^2 s_t^2 (\tilde{m}_{t_1}^2 - \tilde{m}_{t_2}^2) \left( \frac{\tilde{m}_{t_1}^2 - \tilde{m}_{t_2}^2}{\tilde{m}_{t_1}^2 + \tilde{m}_{t_2}^2} \right) + s_t^2 c_b^2 (\tilde{m}_{t_2}^2 - \tilde{m}_{b_1}^2) \left( \frac{\tilde{m}_{t_2}^2 - \tilde{m}_{b_1}^2}{\tilde{m}_{t_2}^2 + \tilde{m}_{b_1}^2} \right) \\ & \left. + s_t^2 s_b^2 (\tilde{m}_{t_2}^2 - \tilde{m}_{b_2}^2) \left( \frac{\tilde{m}_{t_2}^2 - \tilde{m}_{b_2}^2}{\tilde{m}_{t_2}^2 + \tilde{m}_{b_2}^2} \right) - s_b^2 c_b^2 (\tilde{m}_{b_1}^2 - \tilde{m}_{b_2}^2) \left( \frac{\tilde{m}_{b_1}^2 - \tilde{m}_{b_2}^2}{\tilde{m}_{b_1}^2 + \tilde{m}_{b_2}^2} \right) \right\} + O \left[ \left( \frac{\tilde{m}_i^2 - \tilde{m}_j^2}{M_W^2} \right) \left( \frac{\tilde{m}_i^2 - \tilde{m}_j^2}{\tilde{m}_i^2 + \tilde{m}_j^2} \right)^2 \right], \end{aligned} \quad (36)$$

$$\begin{aligned} U_{\tilde{q}} \rightarrow & \sum_{\tilde{q}} \frac{N_c}{12\pi} \left\{ -c_t^2 c_b^2 \left( \frac{\tilde{m}_{t_1}^2 - \tilde{m}_{b_1}^2}{\tilde{m}_{t_1}^2 + \tilde{m}_{b_1}^2} \right)^2 - c_t^2 s_b^2 \left( \frac{\tilde{m}_{t_1}^2 - \tilde{m}_{b_2}^2}{\tilde{m}_{t_1}^2 + \tilde{m}_{b_2}^2} \right)^2 - s_t^2 s_b^2 \left( \frac{\tilde{m}_{t_2}^2 - \tilde{m}_{b_2}^2}{\tilde{m}_{t_2}^2 + \tilde{m}_{b_2}^2} \right)^2 \right. \\ & \left. - s_t^2 c_b^2 \left( \frac{\tilde{m}_{t_2}^2 - \tilde{m}_{b_1}^2}{\tilde{m}_{t_2}^2 + \tilde{m}_{b_1}^2} \right)^2 + c_t^2 s_t^2 \left( \frac{\tilde{m}_{t_1}^2 - \tilde{m}_{t_2}^2}{\tilde{m}_{t_1}^2 + \tilde{m}_{t_2}^2} \right)^2 + s_b^2 c_b^2 \left( \frac{\tilde{m}_{b_1}^2 - \tilde{m}_{b_2}^2}{\tilde{m}_{b_1}^2 + \tilde{m}_{b_2}^2} \right)^2 \right\} + O \left( \frac{\tilde{m}_i^2 - \tilde{m}_j^2}{\tilde{m}_i^2 + \tilde{m}_j^2} \right)^4. \end{aligned} \quad (37)$$

First, we see that in the limit of exact custodial  $SU(2)_V$  symmetry, which corresponds to  $\tilde{m}_{t_1} = \tilde{m}_{b_1} \equiv \tilde{m}_1$ ,  $\tilde{m}_{t_2} = \tilde{m}_{b_2} \equiv \tilde{m}_2$  and  $c_t = c_b \equiv c$ ,  $s_t = s_b \equiv s$ , both  $T_{\tilde{q}}$  and  $U_{\tilde{q}}$  vanish as it is expected, whereas  $S_{\tilde{q}}$  goes as,

$$S_{\tilde{q}} \rightarrow \sum_{\tilde{q}} \frac{N_c}{3\pi} c^2 s^2 \left( \frac{\tilde{m}_1^2 - \tilde{m}_2^2}{\tilde{m}_1^2 + \tilde{m}_2^2} \right)^2 + O \left( \frac{\tilde{m}_1^2 - \tilde{m}_2^2}{\tilde{m}_1^2 + \tilde{m}_2^2} \right)^4 \quad (38)$$

Second, the above formulae show that the decoupling indeed occurs in the three parameters since they go to zero as some power of the proper parameters defined above, which vanish in the infinite masses limit. Besides, the decoupling is much faster in  $U_{\tilde{q}}$  than in  $S_{\tilde{q}}$  and  $T_{\tilde{q}}$ . These results confirm the numerical analyses performed in the literature and agree with the qualitative behaviour discussed in <sup>1,2,3</sup>.

However, we would like to emphasize once more that, contrary to most of the studies in the literature (with the exception of those on  $\Delta\rho$ ), our results are model independent and do not make any reference on whether there is or not a common effective scale of supersymmetry breaking.

Parallel results for the sleptons sector can be obtained by making the corresponding replacements in the above formulae <sup>13</sup>.

## 4.2 Neutralinos and Charginos sector:

Considering together the conditions given for this sector in section 3.2 we get,

$$S_{\tilde{\chi}} = \frac{-1}{3\pi} \log \frac{2\tilde{M}_2^{+2}}{\tilde{M}_3^{o2} + \tilde{M}_4^{o2}}, \quad (39)$$

$$T_{\tilde{\chi}} = \frac{1}{4\pi m_w^2 s_w^2} \left\{ -2(\tilde{M}_1^+ - \tilde{M}_2^o)^2 \log \frac{\tilde{M}_1^{+2} + \tilde{M}_2^{o2}}{2\mu_o^2} - \frac{1}{2}(\tilde{M}_2^+ - \tilde{M}_3^o)^2 \log \frac{\tilde{M}_2^{+2} + \tilde{M}_3^{o2}}{2\mu_o^2} \right. \\ \left. - \frac{1}{2}(\tilde{M}_2^+ - \tilde{M}_4^o)^2 \log \frac{\tilde{M}_2^{+2} + \tilde{M}_4^{o2}}{2\mu_o^2} + \frac{1}{2}(\tilde{M}_3^o - \tilde{M}_4^o)^2 \log \frac{\tilde{M}_3^{o2} + \tilde{M}_4^{o2}}{2\mu_o^2} \right\}, \quad (40)$$

$$U_{\tilde{\chi}} = \frac{4}{3s_w^2} \log \frac{\tilde{M}_2^{o2} + \tilde{M}_1^{+2}}{2\tilde{M}_1^{+2}} + \frac{1}{3s_w^2} \log \left[ \frac{(\tilde{M}_3^{o2} + \tilde{M}_2^{+2})(\tilde{M}_4^{o2} + \tilde{M}_2^{+2})}{2\tilde{M}_2^{+2}(\tilde{M}_3^{o2} + \tilde{M}_4^{o2})} \right], \quad (41)$$

Here the values of the coupling matrices  $O_{L,R}, O'_{L,R}, O''_{L,R}$  corresponding to the large neutralinos and charginos masses limit have been used.<sup>13</sup>

With regard to this sector and by looking at eqs.(39 - 41) we can conclude that, in the large masses limit, the first chargino  $\tilde{\chi}_1^+$  and the two first neutralinos  $\tilde{\chi}_1^o$  and  $\tilde{\chi}_2^o$  decouple completely in the  $S$  parameter. These are precisely the chargino and neutralinos, that in the large masses limit become predominantly gauginos. The decoupling of the other eigenstates  $\tilde{\chi}_2^+, \tilde{\chi}_3^o$  and  $\tilde{\chi}_4^o$  in  $S$  is not evident at a first sight, since it depends on the relative size of the  $\tilde{\chi}_2^+$  mass and the masses of the neutralinos  $\tilde{\chi}_3^o$  and  $\tilde{\chi}_4^o$ . However, we have seen in the above sections, that in the large masses limit, their corresponding squared mass eigenvalues approach to a common value  $\mu^2$  and, in consequence, the decoupling in  $S_{\tilde{\chi}}$  does finally occur. Notice that this result is not model dependent either, since this common value  $\mu^2$  is the unique squared mass parameter that is allowed by supersymmetry to be present at the Lagrangian level and do not depend on the particular assumed SUSY breaking mechanism. Similarly, in the  $T_{\tilde{\chi}}$  and  $U_{\tilde{\chi}}$  parameters the decoupling occurs exactly if the mass eigenvalues in the large mass limit are considered, i.e.,  $\tilde{M}_1^{+2} \rightarrow M_2^2$ ,  $\tilde{M}_2^{+2} \rightarrow \mu^2$ ,  $\tilde{M}_1^{o2} \rightarrow M_1^2$ ,  $\tilde{M}_2^{o2} \rightarrow M_2^2$  and  $\tilde{M}_3^{o2} = \tilde{M}_4^{o2} \rightarrow \mu^2$ .

Finally, we would like to point out that the results for  $S, T$  and  $U$  of the various sectors are finite as they must be and the cancellation of divergences occur between the  $\tilde{t}$  and  $\tilde{b}$  contributions of each generation of squarks, between the  $\tilde{\nu}$  and  $\tilde{\tau}$  contributions of each generation of sleptons and between the charginos and neutralinos.

## 5 Conclusions

The computation of the effective action for the standard particles which results by integrating out all the heavy supersymmetric particles will provide the answer to the question wether the decoupling of heavy supersymmetric particles in the MSSM occurs leading to the SM as the remaining low energy effective theory. In this work we have shown that the contributions from the heavy sparticles to the two point functions part of the effective action can be absorbed into redefinitions of the Standard Model parameters or they are suppressed by inverse powers of the heavy sparticles masses.

More specifically, we have proved analytically that the decoupling of squarks, sleptons, charginos and neutralinos, at one loop level, in the two-points functions of the electroweak gauge bosons takes places. We have considered the limit where the sparticle masses are all large as compared to the  $W^\pm$  and  $Z$  masses and the external momentum. Notice that we have not assumed exact universality of the masses but we have always worked under the plausible assumption that the differences of their squared masses are much smaller than their sums.

Our results for these two-point Green functions in the large SUSY masses limit have been presented analytically and given in terms of the sparticle masses. Therefore, they are general. Namely, they do not depend on the particular choice for the soft-breaking terms. In our opinion, it is more convenient for the analysis of the phenomenon of decoupling to use the physical sparticle masses themselves, being the proper parameters, rather than some other possible mass parameters of the MSSM as, for instance, the  $\mu$ -parameter or the soft-SUSY breaking parameters.

We have shown that the decoupling of sparticles also occurs in the  $S, T$  and  $U$  parameters, and we have presented explicit formulae for these parameters, which illustrate analytically how this decoupling occurs.

Finally, we have explored to what extent the hypothesis of generation of SUSY masses by soft-SUSY breaking terms is relevant for decoupling and we have found instead that the requirement of  $SU(3)_c \times SU(2)_L \times U(1)_Y$  gauge invariance of the explicit mass terms by itself is sufficient to get it.

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