Partially coherent sources with radial coherence

G. PIQUERO[1], M. SANTARSIERO[2,*], R. MARTÍNEZ-HERRERO[1], J. C. G. DE SANDE[3], M. ALONZO[4], AND F. GORI[2]

[1] Departamento de Óptica, Universidad Complutense de Madrid, Ciudad Universitaria, 28040 Madrid, Spain
[2] Dipartimento di Ingegneria, Università Roma Tre, Via V. Volterra 62, 00146 Rome, Italy
[3] ETSIS de Telecomunicación, Universidad Politécnica de Madrid, Campus Sur 28031 Madrid, Spain
[4] Dipartimento SBAI, Università Sapienza, Via A. Scarpa 16, 00161 Roma, Italy
[*] Corresponding author:massimo.santarsiero@uniroma3.it

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Partially coherent sources with radial coherence are proposed. They present a circularly symmetric intensity profile and a degree of coherence whose absolute value only depends on the angular difference between the two considered points. In particular, the source is completely coherent at pairs of points belonging to the same radius. The modal structure of such sources is determined in the general case, and conditions are derived under which the field propagated in paraxial approximation remains radially coherent at any transverse plane. In such cases, the angular dependence of the correlation function is preserved upon propagation, although the intensity profile generally changes. An example of this kind of sources has been experimentally synthesized by means of a simple set up and its coherence characteristics have been tested by means of a Young interferometer.

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The proposal of models that represent partially coherent beams, to be used in the study of their characteristics and the changes they undergo on propagation or through different systems, is a topic that keeps growing. Starting from the first seminal papers [1, 2], numerous works on the structure of scalar partially coherent sources have been published in the literature to date (see, for example, [3–23]). A detailed review on the subject can be found in [24] and [25].

On the other hand, except for the case of sources of the Schell-model type [1], which can be always produced starting from incoherent planar sources with suitable profiles [26], the synthesis of partially coherent sources still remains a critical issue. A greater versatility is offered by those techniques that synthesize partially coherent sources through the superposition of a set of perfectly coherent, but mutually uncorrelated fields. In most cases, such fields occur sequentially across the source plane, and the finite time response of the detector performs their average, as in [27–34].

In this letter we present a class of planar sources having circularly symmetric irradiance and degree of coherence that depends only on the difference between the polar angles of the two considered points, except for a possible phase factor. In a sense, such sources represent the generalization of the ones studied in Refs. [11, 15], where thin annular sources were investigated. Sources we are presenting here are perfectly coherent at pairs of points lying along a radius while, on increasing their angular difference, the coherence can be partial or even vanishing. As we shall see, such a property is not preserved, in general, upon propagation, unless some conditions are imposed to the starting source.

A source of this class has been synthesized by means of a simple optical set up, making use of two independent laser beams and a home-made amplitude hologram, and its coherence properties have been tested by means of a Young interferometer. Differently from most of the techniques used to produce partially coherent sources (not of the Schell-model type), the present one does not require the time integration of a sequence of coherent fields, thus providing a “real-time” synthesis of the source.

A source with radial coherence is characterized by a cross spectral density (CSD) [26] of the form

$$W_0(\mathbf{r}_1,\mathbf{r}_2) = f^*(r_1)f(r_2)\,g(\theta_1-\theta_2), \tag{1}$$

where $\mathbf{r}=(r,\theta)$ is the position vector across the source plane ($z=0$), $f(r)$ is any function, with the only restriction that it vanishes at the origin, and $g(\alpha)$ is a periodic function of α with period 2π. Without loss of generality, we can assume $g(0)=1$. Here and in the following, we omit the explicit dependence on the temporal frequency.

The pertinent irradiance is fully determined by the function $f(r)$, i.e., $W_0(\mathbf{r},\mathbf{r}) = |f(r)|^2$, while the spectral degree of coherence between the points $\mathbf{r}_1$ and $\mathbf{r}_2$, defined as [26]

$$\mu_0(\mathbf{r}_1,\mathbf{r}_2) = \frac{W_0(\mathbf{r}_1,\mathbf{r}_2)}{\sqrt{W_0(\mathbf{r}_1,\mathbf{r}_1)\,W_0(\mathbf{r}_2,\mathbf{r}_2)}}, \tag{2}$$

turns out to be

$$\mu_0(\mathbf{r}_1,\mathbf{r}_2) = \frac{f^*(r_1)f(r_2)}{|f(r_1)f(r_2)|}\,g\,(\theta_1-\theta_2)\,. \tag{3}$$

Therefore, up to a possible phase factor, the function $g(\theta_1-\theta_2)$ gives the degree of spectral coherence, which is independent of the radial coordinates of the two points.

Due to its periodicity, the function $g(\alpha)$ can be expressed as the following Fourier series:

$$g(\alpha)=\sum_{n=-\infty}^{\infty} g_n\, e^{in\alpha}, \tag{4}$$

with

$$g_n=\frac{1}{2\pi}\int_0^{2\pi} g(\alpha)\, e^{-in\alpha}\, d\alpha\,. \tag{5}$$

Since g has to be Hermitian, the Fourier coefficients g_n are real. Furthermore, it turns out [10, 12] that necessary and sufficient condition for the CSD to be bona fide is that all g_n be nonnegative.

To study the coherence features of the field radiated from a source with radial coherence, Eq. (4) is of help. In fact, on inserting Eq. (4) into Eq. (1) we find

$$W_0(\mathbf{r}_1,\mathbf{r}_2)=\sum_{n=-\infty}^{\infty} g_n\,\psi_n^*(\mathbf{r}_1;0)\psi_n(\mathbf{r}_2;0)\,, \tag{6}$$

with

$$\psi_n(\mathbf{r};0)=f(r)\,e^{-in\theta}\,. \tag{7}$$

Since the functions $\psi_n(\mathbf{r};0)$ are mutually orthogonal on the plane $z=0$ and the weights g_n are nonnegative, the expression in Eq. (6) can be read as the modal expansion [26] of the CSD in Eq. (1). This corresponds to consider the partially coherent source as the superposition of a set of suitably weighted, perfectly coherent and mutually uncorrelated modes. Therefore, the CSD at a distance z from the source can be evaluated from the expression of the propagated modes as

$$W_z(\mathbf{R}_1,\mathbf{R}_2)=\sum_{n=-\infty}^{\infty} g_n\,\psi_n^*(\mathbf{R}_1;z)\psi_n(\mathbf{R}_2;z)\,, \tag{8}$$

where $\mathbf{R}_1=(R_1,\phi_1)$ and $\mathbf{R}_2=(R_2,\phi_2)$ are the position vectors of two points across the plane $z=$ constant, and the propagated modes, in paraxial conditions, are

$$\psi_n(\mathbf{R};z)=-\frac{ik\,e^{ikz}}{2\pi z}\int \psi_n(\mathbf{r};0)\exp\left(\frac{ik}{2z}\left|\mathbf{R}-\mathbf{r}\right|^2\right)d\mathbf{r}\,. \tag{9}$$

where k is the wavenumber.

In conclusion, the following expression for the CSD across the plane $z=$constant is obtained:

$$W_z(\mathbf{R}_1,\mathbf{R}_2)=\frac{k^2}{z^2}\exp\left[\frac{ik}{2z}\left({R_2}^2-{R_1}^2\right)\right] \times\sum_{n=-\infty}^{\infty} g_n\,F_n^*(R_1;z)\,F_n(R_2;z)\,\exp\left[i\,n\,(\phi_1-\phi_2)\right], \tag{10}$$

with

$$F_n(R;z)=\int_0^{\infty} f(r)\exp\left(\frac{i\,k}{2z}r^2\right)J_n\left(\frac{kRr}{z}\right)r\,dr, \tag{11}$$

being $J_n(\cdot)$ the Bessel function of the first kind and order n [35].

Equation (10) provides the expression of the CSD of the field propagated from any partially coherent source with radial coherence. Apparently, such CSD still depends on the angular coordinates through their difference, i.e., $\Delta\phi=\phi_1-\phi_2$, but in general has not the form required for a radially coherent source [Eq. (1)].

A significant exception occurs when only two terms in the expansion are nonvanishing, namely, for $n=\pm m$, with $m>0$. In such a case, in fact, due to the symmetry properties of the Bessel functions [35], the following relation holds:

$$F_{-m}(R;z)=(-1)^m F_m(R;z)\,, \tag{12}$$

and Eq. (10) becomes

$$W_z(\mathbf{R}_1,\mathbf{R}_2)=\frac{k^2}{z^2}\exp\left[\frac{ik}{2z}\left({R_2}^2-{R_1}^2\right)\right] \times F_m^*(R_1;z)\,F_m(R_2;z)\,g(\phi_1-\phi_2)\,, \tag{13}$$

with

$$g(\alpha)=g_m e^{im\alpha}+g_{-m}e^{-im\alpha}\,. \tag{14}$$

Therefore, the expression of the degree of coherence does not change on propagation, even though the intensity profile generally changes. Furthermore, due to the presence of only two modes, having vortices with opposite charge, it turns out that the resulting beams carry orbital angular momentum [36] proportional to $m\,(g_m-g_{-m})$.

The condition $g(0)=g_m+g_{-m}=1$ allows us to write the nonnegative coefficients $g_{\pm m}$ as $g_m=\cos^2\eta$ and $g_{-m}=\sin^2\eta$, for a given real parameter η, so that $g(\alpha)$ takes the form

$$g(\alpha)=\cos(m\alpha)+i\,\delta\,\sin(m\alpha)\,, \tag{15}$$

with $\delta=(g_m-g_{-m})=\cos(2\eta)$. The modulus of the degree of coherence then reads

$$|\mu_z(\mathbf{R}_1,\mathbf{R}_2)|^2=\cos^2[m(\phi_1-\phi_2)]+\delta^2\sin^2[m(\phi_1-\phi_2)]\,. \tag{16}$$

Across any transverse plane, the propagated field is perfectly coherent at points lying along the same radius, but perfect coherence is also obtained when $|\Delta\phi|$ equals a multiple of π/m. The lowest coherence is reached when $|\Delta\phi|$ is an odd multiple of $\pi/2m$, in which case $|\mu_z|=|\delta|$. The complete incoherence occurs for these values of $|\Delta\phi|$ and $\delta=0$, i.e., when $g_m=g_{-m}=1/2$.

A stricter condition is found if one requires that not only the angular dependence of the CSD remains unchanged upon propagation, but also that its radial part is invariant in shape. Indeed, this happens when the two nonvanishing modes of the source are of the Laguerre–Gaussian type [37], i.e., when

$$\psi_{\pm m}(\mathbf{r};0)\propto L_\ell^m\left(\frac{2r^2}{w_0^2}\right)\left(\frac{\sqrt{2}\,r}{w_0}\right)^m e^{-r^2/w_0^2}\,e^{\mp im\theta}\,, \tag{17}$$

for any choice of the waist size w_0 and of the index ℓ, where L_ℓ^m is the generalized Laguerre polynomial [35]. In such a case, in fact, the two modes keep their initial form during propagation, except for common curvature and transverse scale factors [37]. They also acquire a Gouy phase depending on the mode indices, that cancels when the CSD is evaluated [see Eq. (8)]. As a consequence, the only effects of the propagation are a transverse widening of the CSD and the appearance of a spherical curvature term. The CSD is then said to be shape invariant. Note that, once w_0 has been fixed, the shape invariance of the CSD is guaranteed for any choice of ℓ in Eq. (17), so that a whole class of shape-invariant CSDs can be found with the same angular dependence. Sources of the latter kind were studied in [6] and, more recently, in [30, 38–41], in connection with their property of giving rise to beams carrying orbital angular momentum.

A simple technique can be used to synthesize partially coherent sources with radial coherence, for any possible function $f(r)$ and for $g(\alpha)$ given by Eq. (15). Let us consider, on the plane $z = 0$, the superposition of two perfectly coherent but mutually independent light fields having vortices of opposite charges. A typical realization of the resulting field will be of the form

$$V_0(\mathbf{r}) = A_1 f(r) e^{\mathrm{i}m\theta} + A_2 f(r) e^{-\mathrm{i}m\theta}, \quad \textbf{(18)}$$

with A_i $(i = 1, 2)$ being random variables such that $\langle A_1^* A_2 \rangle = 0$. The intensities of the two beams are chosen as $\langle |A_1|^2 \rangle = I_0\, g_m$ and $\langle |A_2|^2 \rangle = I_0\, g_{-m}$, respectively, where I_0 is a constant intensity value. The pertaining CSD function is then given by

$$W_0(\mathbf{r}_1, \mathbf{r}_2) = \langle V_0^*(\mathbf{r}_1) V_0(\mathbf{r}_2) \rangle = I_0\, f^*(r_1) f(r_2)\, g(\theta_1 - \theta_2)\ , \quad \textbf{(19)}$$

which is exactly of the form required by Eqs. (1) and (15).

A way to implement such a superposition could be through the use of two independent laser beams whose transverse profiles are modulated by suitable amplitude filters and spiral phase plates or, alternatively, by two spatial light modulators, and eventually combined by mean of a beamsplitter.

A much simpler and cheaper approach can be adopted when the function f in the CSD to be synthesized is real. It consists in using two independent laser beams impinging onto a suitably designed hologram. The transmission function of the hologram, τ, reproduces the intensity pattern obtained from the interference of a tilted plane wave with the field $f(r)\exp(\mathrm{i}m\theta)$, that is,

$$\begin{aligned} \tau(\mathbf{r}) &= \left| e^{\mathrm{i}k_x x} + f(r)\, e^{\mathrm{i}m\theta} \right|^2 \\ &= 1 + f^2(r) + f(r)\, e^{\mathrm{i}k_x x}\, e^{-\mathrm{i}m\theta} + f(r)\, e^{-\mathrm{i}k_x x}\, e^{\mathrm{i}m\theta}\ . \end{aligned} \quad \textbf{(20)}$$

where k_x is the x-component of the plane wave and is related to its tilt. In particular, the field used in our experiment is a Laguerre–Gaussian mode across its waist, with $\ell = 0$ and $m = 1$, and $k_x w_0 = 100$. The resulting transmittance (T) is shown in the inset of Fig. 1.

When this transparency (placed at the plane $z = 0$) is illuminated by a plane wave of amplitude A_1 and wave vector $(k_x, 0, k_z)$, the transmitted field, at $z = 0$, is

$$\begin{aligned} V_+(\mathbf{r}) = \ & A_1\, e^{\mathrm{i}k_x x} \tau(\mathbf{r}) = A_1\, \left[1 + f^2(r)\right]\, e^{\mathrm{i}k_x x} \\ & + A_1\, f(r) e^{2\mathrm{i}k_x x}\, e^{-\mathrm{i}m\theta} + A_1\, f(r)\, e^{\mathrm{i}m\theta}\ . \end{aligned} \quad \textbf{(21)}$$

On the other hand, when the same transparency is illuminated by another plane wave, having wave vector $(-k_x, 0, k_z)$ and amplitude A_2, the transmitted field is

$$\begin{aligned} V_-(\mathbf{r}) = \ & A_2\, e^{-\mathrm{i}k_x x} \tau(\mathbf{r}) = A_2\, \left[1 + f^2(r)\right]\, e^{-\mathrm{i}k_x x} \\ & + A_2\, f(r) e^{-\mathrm{i}m\theta} + A_2 f(r) e^{-2\mathrm{i}k_x x}\, e^{\mathrm{i}m\theta}\ . \end{aligned} \quad \textbf{(22)}$$

If the above plane waves impinge onto the hologram simultaneously, among the contributions to the output field the only ones that propagate along the z axis give rise to a field coincident with that in Eq. (18). As a consequence, the CSD in Eq. (19) is synthesized when A_1 and A_2 are mutually uncorrelated and $\langle |A_1|^2 \rangle = I_0\, g_m$ and $\langle |A_2|^2 \rangle = I_0\, g_{-m}$.

The optical set up used to implement the above approach is shown in Fig. 1. Two independent He-Ne laser beams ($\lambda = 633$ nm) are spatially filtered through the pinholes PH_1 and PH_2, expanded, and sent onto the transparency T, having transmittance τ. The power balancing of the two beams can be tuned by means

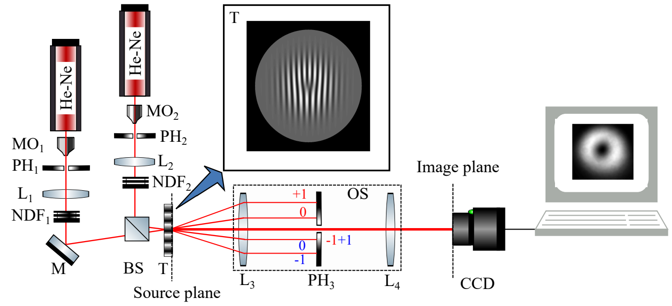

Fig. 1. Experimental setup for the synthesis of partially radially coherent sources. MO: microscope objectives; PH: pinholes; M: mirror; NDF: neutral density filters; BS: beam splitter; T: transparency; L: lenses.

of variable neutral density filters (NDF), while their propagation directions can be set as the prescribed ones through the mirror M and the beam splitter BS. Note that tuning the power balancing of the two input beams allows the generation of partially coherent beams with variable orbital angular momentum.

In our experiment, the hologram was realized taking a picture of the complement of the profile in Eq. (20), displayed on a computer screen, using an analog camera with a b/w negative film. The obtained transparency corresponds to the profile of T shown in Fig. 1, with $w_0 = 0.6$ mm. NDF_1 and NDF_2 were set in such a way that the two impinging beams had the same intensity on the hologram, so that a source with $\delta = 0$ was synthesized. The intensity profile of the field emerging from the hologram is

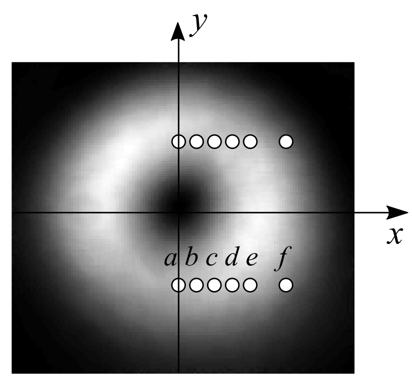

Fig. 2. Positions of the Young's hole pairs across the source plane. For each pair, the holes are placed along y axis.

imaged onto the detector of a CCD camera by means of the $4f$ optical system OS, consisting of lenses L_3 and L_4. Pinhole PH_3 selects the field component propagating along the z-axis. The recorded source intensity is shown in Fig. (2).

To test the coherence properties of the source we used a Young mask, consisting of an opaque screen with two pinholes with 0.3 mm diameter, 2 mm apart. The mask was placed across the image plane of OS, and far-field interference fringes were observed at the output plane of a $2f$ optical system (not shown in Fig. 1). The mask could be translated across the transverse plane and rotated around the z-axis. In all our measurements, the Young holes were aligned along the vertical axis (y) while their position across the plane was changed to detect the coherence between points at different angular separations. It is worth mentioning that analogous information could have been obtained by employing a modified Young mask, namely, the double angular slit, already used to analyze the angular coherence of a mixture of Laguerre-Gaussian modes [39–41].

Starting from the position denoted by (a) in Fig. 2, for which the two pinholes are placed on opposite positions with respect to the beam center ($\Delta\theta = \pi$), the mask was displaced along the x-axis to positions (b)-(f) (in steps of 0.25 mm except from (e) to (f) where the step is 0.5 mm), in such a way that the angular separation between the holes decreased from π up to the minimum value of $\simeq 0.31\pi$. Interference fringes at the output of the Young interferometer, recorded by the CCD camera, are shown in Figs. 3(a)-(f). It can be noted that, in accordance with the theoretical predictions, the visibility is maximum when $\Delta\theta = \pi$, decreases for successive positions from (b) to (d), and almost vanishes for $\Delta\theta = \pi/2$ (e). Then, it increases again for angular differences lower than $\pi/2$ (f).

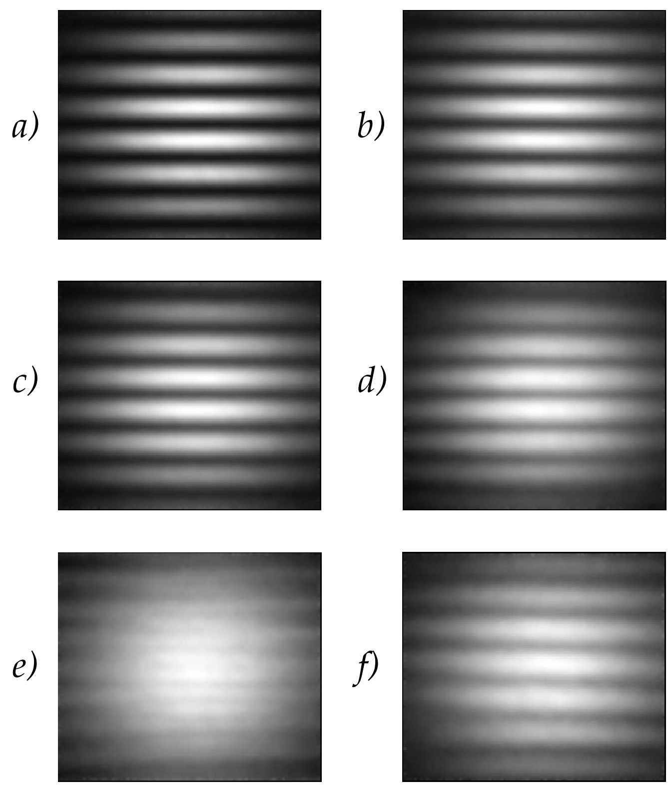

Fig. 3. Experimental interference patterns obtained when the Young holes are set at the positions shown in Fig. 2.

In conclusion, partially coherent sources with radial coherence have been introduced, for which the degree of coherence between two points depends only on their angular separation. Analytical expressions for the CSD of the propagated field have been derived, and conditions under which the degree of coherence remains invariant during paraxial propagation have been obtained. A very simple optical setup has been designed and implemented to experimentally generate in real time beams with propagation-invariant radial coherence and tunable orbital angular momentum. The coherence properties of the synthesized source have been tested by means of a Young interferometer, showing good agreement with the theoretical predictions.

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