Why do variance swaps exist?

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Abstract

This paper studies the determinants of the variance risk premium and concludes on the hedging possibilities offered by variance swaps. We start by showing that the variance risk premium responds to changes in higher order moments of the distribution of market returns. But the uncertainty that determines the variance risk premium –the fear by investors to deviations from Normality in returns- is also strongly related to a variety of risks: risk of default, employment growth risk, consumption growth risk, stock market risk and market illiquidity risk. Therefore, the variance risk premium could be interpreted as reflecting the market willingness to pay for hedging against financial and macroeconomic sources of risk. We provide additional evidence in support of that view.

1. Introduction

Why is the variance risk premium reported to be negative, on average, for all available horizons? The main objective of this paper is precisely to answer this question. Since the payoff of a variance swap contract is the difference between the realized variance and the variance swap rate, negative returns to long positions on variance swap contracts for all time horizons mean that investors are willing to accept negative returns for purchasing realized variance. Equivalently, investors who are sellers of variance and are providing insurance to the market, require substantial positive returns. This may be rational since the correlation between volatility shocks and market returns is known to be strongly negative and investors want protection against stock market crashes. The crucial issue is how large a premium should be charged for offering that hedge. In terms of variance swaps, the key challenge is to formally explain the large average negative variance risk premium observed at all horizons.

In this paper, we follow the theoretical model proposed by Chabi-Yo (2009) to find evidence that the variance risk premium responds to changes in higher order moments of the conditional distribution of market returns over and above the mean and variance of the stock market portfolio. Our findings suggest that the variance swap is a financial instrument that offers hedging against time variation and non-Normality in the conditional distribution of returns. The issue then becomes to identify the economic sources of non-Normality of market returns. In that sense, we provide evidence that the same determinants of the variance risk premium are also able to explain standard economic risks, such as equity market risk, aggregate default risk, market-wide illiquidity, and consumption and employment growth risks. The existence of common determinants of the variance risk premium and indicators of different types of risk suggests that variance swaps may offer coverage against them. Indeed, we find that going long in the variance swap contract provides a hedge against equity market risks, as well as against interest rate risks and business cycle risks. Hence the variance risk premium can be interpreted as reflecting the market willingness to pay for hedging against these financial and macroeconomic risks.

Since our analysis suggests that variance swaps may be effective against risks other than market risk, we search for additional evidence in favor of variance swaps being a significant asset for portfolio risk management. To that end, we analyze whether variance swap contracts are redundant assets, relative to standard benchmarks, by performing the step-down spanning tests in Kan and Zhou (2008). Our implementation of these spanning tests compares the minimum variance frontier associated to a universe of four benchmark US assets, namely, the S&P500 returns, the Aaa and Baa corporate bond yields, and the 10-year government bond yield, with the minimum variance frontier of the expanded set which additionally includes the excess return of the variance swap contract, which we take as the variance risk premium. We systematically reject spanning, suggesting that the variance risk premium contains incremental relevant information not included in the four benchmark assets. The reason seems to be related to the diversification opportunities generated by the variance swaps given the negative correlation between the payoff of the swap and the payoff of standard assets. The analysis for different maturities of the swap contracts reveals that, at the longest horizons, the improvement of the minimum variance frontier comes primarily from the tangency portfolio while, at the shortest horizons, the strong evidence of improving the investment opportunity set comes from the global minimum portfolio rather than from the tangency portfolio. In any case, and for all types of tests, we always reject spanning.

This paper is organized as follows. Section 2 briefly describes the variance swap contract and defines the variance risk premium, while Section 3 contains a description of the data. The determinants of the variance risk premium, and the relationship between these determinants and several financial and economic risks, is discussed in Section 4. The hedging ability of the variance risk premium against a variety of financial and economic risks is reported in Section 5. Section 6 provides the results from mean-variance spanning tests, and Section 7 concludes with a summary of our findings.

2. Variance Swap Contracts and the Variance Risk Premium

A variance swap is an over-the-counter financial instrument that pays the difference between a standard estimate of the realized variance of the return on a given asset and the fixed variance swap rate. More in detail, one leg of the variance swap pays an amount based upon the realized variance of daily log returns, computed with the commonly used closing price of the underlying asset. The other leg of the swap pays a fixed amount, the strike, quoted at the deal's inception. Thus the net payoff to the counterparties is the difference between these two values. It is settled in cash at the expiration of the deal, though some cash payments are likely to be made along the way by one or the other counterparty to maintain an agreed upon margin. The payoff of a variance swap is therefore given by,

$$N_{var}\left(RV_{t,t+\tau} - SW_{t,t+\tau}\right), \tag{1}$$

where N_{var} denotes variance notional, also called variance units, $RV_{t,t+\tau}$ is the annualized realized variance over the life of the contract, and $SW_{t,t+\tau}$ is the delivery price quoted at time t for the variance, also known as the variance swap rate with maturity at $t+\tau$.

Since variance swaps cost zero at entry, no arbitrage requires that the variance swap rate must be equal to the risk-neutral expected value of the realized variance,

$$SW_{t,t+\tau} = E_t^Q\left(RV_{t,t+\tau}\right), \tag{2}$$

where $E_t^Q(.)$ is the time-t conditional expectation operator under some risk-neutral measure Q. The variance risk premium at period t is then defined as,

$$VRP_t^{t+\tau} = E_t^P\left(RV_{t,t+\tau}\right) - SW_{t,t+\tau}, \tag{3}$$

where $E_t^P(.)$ is the time-t conditional expectation operator under the physical probability measure P. If investors price variance risk, the variance swap rate will differ from the expected realized variance under P at the corresponding horizon, the difference being the variance risk premium.

3. Data and Descriptive Statistics

In this paper we analyze variance swap contracts on the S&P 500 index. Daily variance swap rates on five different maturities from January 4, 1996 to January 31, 2007 were obtained from the Bank of America. We get monthly data by using the quotes on the last day of each month. Our estimation of the realized variance employs intra-daily returns on the S&P 500 index observed at 30-minute intervals, from 9 a.m. to 3 p.m. For each month in our sample, we compute the realized variance for each maturity τ of a variance swap contract ($\tau = 1$, 2, 3, 6, and 12 months) using quadratic changes on the value of the S&P 500 index,

$$RV_{t,t+\tau} = L \sum_{l=1}^{L} \left(\frac{P_{t,l} - P_{t,l-1}}{P_{t,l-1}} \right)^2 , \tag{4}$$

where L is the number of 30-minute intervals comprised in the interval $(t, t+\tau)$. We work with variance swap rates and realized variances in percent numbers.

For each month t and each maturity τ, we compute the variance risk premium, *VRP*, as the difference between the realized variance and the swap rate,

$$VRP_{t,t+\tau} = RV_{t,t+\tau} - SW_{t,t+\tau} . \tag{5}$$

Some of our tests also employ the log return for being long in the variance swap contract. Then, we also denote

$$vrp_{t,t+\tau} = \log\left(\frac{RV_{t,t+\tau}}{SW_{t,t+\tau}} \right). \tag{6}$$

Clearly, in both cases the variance risk premium is only known at time $t+\tau$, since the realized variance is only observed at the end of the swap contract.

Figure 1 displays variance swap rates and realized variance for 1-, 3- and 6-month maturities. As expected, the swap rate is most often above the level of realized variance, especially for longer maturities. This evidence is similar to that shown by Carr and Wu (2009) for stock market indices and, to a lesser extent, for individual stocks.[1] It is clear that investors are willing to accept a significantly negative return to long variance swaps on the S&P index in exchange for being hedged against future unexpected volatility shocks. Therefore, shorting variance swap contracts in the S&P index generates significantly positive average excess returns during our sample period, since the variance risk premium can be seen as the return on holding the variance swap

[1] Driessen, Maenhout, and Vilkov (2009), and Vilkov (2008) show that the variance risk premium for stock indices are systematically larger, i.e., more negative, than for individual securities. They argue that the variance risk premium can in fact be interpreted as the price of time-varying correlation risk.

contract. Panel A of Table 1 reports descriptive statistics of the $VRP_{t,t+\tau}$ for alternative maturities. The variance risk premium is always negative on average, and it becomes more negative with maturity. Panel B of Table 1 reports the correlation coefficients between the variance risk premia at any two different maturities. Correlations between variance risk premia at adjacent maturities are high, debilitating for faraway maturities. The correlation matrix suggests the existence of at least two factors in the structure of variance risk premium.[2]

We obtain nominal consumption expenditures on nondurable goods and services from NIPA Table 2.8.5. Population data is taken from NIPA Table 2.6, and the price deflator is computed using prices from NIPA Table 2.8.4 with basis on year 2000. All this information is used to construct monthly seasonally adjusted real per capita consumption expenditures on nondurable goods and services. Seasonally adjusted monthly data on the number of employees is obtained from the Bureau of Labor Statistics. Then, monthly series of cumulative growth rates for the five maturity intervals $(t, t+\tau)$ are computed for non-durable consumption and services, as well as for the number of employees.

Stock market data is taken from Kenneth French´s web page. Monthly data on value-weighted stock market portfolio returns (R_W) and the risk-free rate (R_f) were deflated using the consumption price deflator. We also collect the size and value Fama-French risk factors (*SMB* and *HML*). Price-dividend ratio in logs (*PD*) is computed from the original series in Robert Shiller´s web page. Additionally, yields for the 10-year Government Bond, the 1-month T-Bill, and the Moody's Baa Corporate Bond have been obtained from the Federal Reserve Statistical Release.

We compute three state variables based on interest rates. R_f *STATE* is the risk-free rate after having subtracted its average over the last twelve months as a measure of trend. *TERM* is a term structure slope, computed as the difference between the 10-year Government Bond and 1-month T-Bill yields. *DEFAULT* is the difference between Moody´s yield on Baa Corporate Bonds and the 10-year Government Bond yields. We

[2] This is consistent with the formal analysis contained in Egloff, Leippold, and Wu (2007), and Amengual (2009). They show that two factors are needed to capture the term structure variation of the variance swap rates. The first factor controls the instantaneous variance rate variation, while the second represents the level to which the variance reverts. Todorov (2009) allows for both stochastic volatility and jumps to be reflected in the variance risk premium.

compute monthly series of cumulative returns corresponding to the five maturity intervals of the variance swap rates for the market return, the risk-free rate, the three Fama-French factors, and R_f *STATE*. We also compute innovations corresponding to the five maturity intervals for the price-dividend ratio and the *TERM* and *DEFAULT* variables as the residual in a regression of each variable at month $t+\tau$ on the observation at month *t*.[3]

Finally, we also use a market-wide illiquidity indicator based on the aggregate illiquidity ratio proposed by Amihud (2002),[4] as the ratio of the absolute daily return over the dollar volume for a given stock, which is closely related to the notion of price impact, $Illiq_{j,d} = \frac{|R_{j,d}|}{DVol_{j,d}}$, where $|R_{j,d}|$ is the absolute return of asset *j* on day *d*, and $DVol_{j,d}$ is the dollar volume of asset *j* during day *d*. This measure is averaged monthly and across all *N* available stocks to obtain the market-wide illiquidity measure for each month *t*,

$$Illiq_{m,t} = \frac{1}{N}\sum_{j=1}^{N}\left(\frac{1}{D_{j,t}}\sum_{d=1}^{D_{j,t}} Illiq_{j,d}\right), \tag{7}$$

where $D_{j,t}$ is the number of days for which data about stock *j* are available in month *t*.[5] As with previous variables, a measure of market illiquidity innovations was obtained as the residual from a regression of $Illiq_{m,t+\tau}$ on $Illiq_{m,t}$.[6]

[3] Similarly, R_f *STATE* can be interpreted as the innovation in the risk-free interest rate.

[4] The main advantage of Amihud´s illiquidity ratio is that it can be easily computed using daily data during long periods of time. Moreover, Hasbrouck (2009) shows that, at least for US data, Amihud´s ratio better approximates Kyle´s lambda relative to competing measures of illiquidity.

[5] We use daily data from CSRP on all individual stocks with at least 15 observations for the ratio within the considered month, except for September 2001, when we just required 12 observations.

[6] To have numerical values closely resembling units of rates of returns, the residuals of the illiquidity measure are standardized by dividing by ten times their sample standard deviation and adding one. See Márquez, Nieto, and Rubio (2009) for further details.

4. The Determinants of the Variance Risk Premium, Non-Normality and Economic Risks

To interpret the large negative magnitude of the variance risk premium across different horizons we employ the pricing model recently proposed by Chabi-Yo (2009). He obtains a stochastic discount factor in which coskewness and the market volatility risk factors are endogenously determined. His model is an extension of the coskewness models of Rubinstein (1973), Kraus and Litzenberger (1976), and Harvey and Siddique (2000) in which the expected risk premium for any stock is determined not only by coskewness but also by the co-movement between the market volatility and the return on the stock. Also this pricing expression explicitly depends on the cross-sectional average of investor risk tolerances and on the weighted average of investor skewness preferences.

An implication of the Chabi-Yo's asset pricing model, especially relevant for our purposes, is that negative skewness and high excess kurtosis, together with a high level of preference for skewness are the two main sources of negative variance risk premium. Moreover, as long as the skewness preference parameter is higher than one, a high correlation of the market volatility with the squared market return generates an even more negative variance risk premium. Under this model, the variance risk premium is given by,

$$VRP_{t,t+\tau} = \lambda_0 + \lambda_W \left(\sigma_{Wt,t+\tau} S_{Wt,t+\tau} \right) + \lambda_{SKD} \left(\sigma_{Wt,t+\tau} (K_{Wt,t+\tau} - 1) \right) + \lambda_{VOL} \nu_{Wt,t+\tau} + \varepsilon_{t,t+\tau} \tag{8}$$

where σ_W, S_W, K_W represent the conditional standard deviation, conditional skewness, and conditional kurtosis of the market return respectively, computed over the time interval described by the subindices, $\nu_{Wt,t+\tau} = Cov_t\left(\sigma^2_{Wt,t+\tau}, R^2_{Wt,t+\tau}\right) / Var_t\left(\sigma^2_{Wt,t+\tau}\right)$ and $\lambda_W > 0$, $\lambda_{SKD} < 0$ and $\lambda_{VOL} < 0$.

Results from the estimation of equation (8) are reported in Table 2.[7] We use two alternative measures for the moments entering as independent variables. First, we calculate realized volatility, skewness and kurtosis from 30-minute intra-daily data

[7] In order to reduce space, and for all the tests of this section (Tables 2, 3 and 4), we only provide results regarding three swap maturities, 1, 6 and 12 months. The results related to the other two horizons are available upon request.

between 9 a.m. to 3 p.m. on S&P 500 index returns for the time interval defined by each swap maturity. Estimation results related to these sample (unconditional) moments are reported in Panel A of Table 2.

Alternatively, since moments in equation (8) are in fact conditional moments, we follow the approach in León, Rubio, and Serna (2005) to estimate conditional variance, skewness, and kurtosis. The authors suggest estimating a Gram-Charlier series expansion of the Normal density function for the return innovation. Their model is given by,

$$\begin{aligned} &R_{Wt} = E_{t-1}(R_{Wt}) + e_t \quad , \; e_t \approx \left(0, \sigma_e^2\right) \\ &e_t = \sigma_{Wt}\eta_t \, , \; \eta_t \, (0,1) , \quad e_t \, / \, I_{t-1} \approx \left(0, \sigma_{Wt}^2\right) \\ &\sigma_{Wt}^2 = \beta_0 + \beta_1 e_{t-1}^2 + \beta_2 \sigma_{Wt-1}^2 \\ &S_{Wt} = \gamma_0 + \gamma_1 \eta_{t-1}^3 + \gamma_2 S_{Wt-1} \\ &K_{Wt} = \delta_0 + \delta_1 \eta_{t-1}^4 + \delta_2 K_{Wt-1} \end{aligned} \tag{9}$$

It must be noted that S_{Wt}, K_{Wt} are now the conditional moments of the standardized residual $\eta_t = e_t / \sigma_{Wt}$. The Gram-Charlier series expansion of the Normal density function for the standardized innovation, truncated at the fourth moment is,

$$g\left(\eta_t \middle| I_{t-1}\right) = \phi(\eta_t)\left[1 + \frac{S_{Wt}}{3!}\left(\eta_t^3 - 3\eta_t\right) + \frac{K_{Wt} - 3}{4!}\left(\eta_t^4 - 6\eta_t^2 + 3\right)\right] = \phi(\eta_t)\Psi(\eta_t) , \tag{10}$$

where $\phi(\eta_t)$ denotes the standard Normal probability density function, while $\Psi(\eta_t)$ denotes the fourth order polynomial in brackets in (10). As in León, Rubio, and Serna (2005), we follow the suggestion in Gallant and Tauchen (1989) to transform the expression (10) into an actual density function by defining $f\left(\eta_t \middle| I_{t-1}\right) = \dfrac{\phi(\eta_t)\Psi^2(\eta_t)}{\Gamma_t}$

where $\Gamma_t = 1 + \dfrac{S_{Wt}^2}{3!} + \dfrac{(K_{Wt} - 3)^2}{4!}$ is the integral of $g\left(\eta_t \middle| I_{t-1}\right)$ over $\Re^5$. The resulting function is everywhere positive and integrates to one. Hence, except by constants, the log-likelihood function for each observation from the conditional distribution for $e_t = \sigma_{Wt}\eta_t$, is given by,[8]

$$l_t = -\frac{1}{2} \ln \sigma_{Wt}^2 - \frac{1}{2}\eta_t^2 + \ln\left(\Psi_t^2(\eta_t)\right) - \ln \Gamma_t \qquad (11)$$

Estimation results using these conditional moments are reported in Panel B of Table 2.

Panel A of Table 2 reports OLS estimates from equation (8), autocorrelation-robust standard errors in parenthesis, and the *R*-square for three different maturities of the variance swaps: 1 month, 6 months and 12 months. The overall fir of the model increases with the maturity, as the *R*-squared statistics indicate. Regarding the individual estimated coefficients, we first note that, at the 1- and 6-month horizons, the cross product of volatility and kurtosis is the only variable with a statistically significant coefficient and the negative expected sign. Other things equal, as more volatility uncertainty is expected in the market in the form of higher kurtosis, the variance swap rate becomes higher and the variance risk premium more negative. At the shortest horizon, the coefficient associated with the cross product of volatility and skewness is estimated with very little precision. As the time horizon increases, the estimated coefficient in this cross product increases drastically although it is not estimated with precision. On the other hand, the estimated effect of the cross product of volatility and kurtosis is quite stable but a loss of precision weakens its statistical significance at the longest horizon.

Panel B of Table 2 provides the estimation results from equation (8) using conditional moments. Unfortunately, in this case, the cross products that constitute the independent variables in equation (8), which are constructed as part of the estimation procedure, turn out to be strongly and negatively correlated, which precludes us from analyzing in detail the estimates of individual coefficients since they lack the required precision to be safely interpreted. For this reason, we only provide *R*-squared statistics. The use of the conditional moments estimates produces much higher *R*-squared coefficients in the estimation of equation (8); it turns out that the cross products of

8 To reduce numerical problems when estimating (9), we restrict the constant terms in the equations for each of the three conditional moments to take their long-run values.

conditional volatility time skewness ($\sigma_{Wt} S_{Wt}$) and kurtosis ($\sigma_{Wt}(K_{Wt} - 1)$) explain approximately 30 percent of the variability of the variance risk premium at the different horizons.

The overall evidence of Table 2 suggests that the variance risk premium may be generated by the desire of investors to hedge against deviations from Normality in the higher order moments of the distribution of returns. Then, it seems natural to ask whether the fears that make the variance risk premium to be negative and high –the fear to deviations from Normality- are also related to standard measures of financial and macroeconomic risks. To pursue this analysis we now estimate the following regression

$$Y_{t,t+\tau} = \lambda_0 + \lambda_W \left(\sigma_{Wt,t+\tau} S_{Wt,t+\tau}\right) + \lambda_{SKD} \left(\sigma_{Wt,t+\tau}(K_{Wt,t+\tau} - 1)\right) + \lambda_{VOL} v_{Wt,t+\tau} + \mu_{t,t+\tau} \tag{12}$$

where the dependent variable (Y) represents a specific type of economic or financial risk. We consider different state variables grouped into three kinds of risk: equity market risk, interest rate risk, and business cycle risk. The first group of variables contains the three Fama-French (1993) factors $\left(R_W - R_f, SMB, HML\right)$ and the innovation in the price-dividend ratio (*PD*). In the second group we consider three variables related to the interest rate risk: the fluctuations in the detrended level of the risk-free real interest rate $\left(R_f STATE\right)$, the surprises in the slope of the yield curve (*TERM*), and the innovations in the default premium (*DEFAULT*). Finally, we use the growth rate of per capita real aggregate non-durable consumption, the total employment growth rate, and the innovations in the market-wide illiquidity measure as business cycle indicators.

Results from the estimation of equation (12) are presented in Table 3. Consistently with Table 2, it has two panels. Estimates in Panel A refer to regressions using unconditional moments while Panel B report results from regressions employing conditional moments. In this second case, we only provide the *R*-squared statistic of each regression because of the co-linearity problems mentioned above.

Table 3 shows that, generally speaking and for the two panels, all risk indicators present low explanatory power at the shortest horizon, but this overall fit increases substantially with the time horizon. To mention a few, the *R*-squared statistic ranges from 0.109 to 0.577 for default risk in Panel A, and from 0.238 to 0.517 for illiquidity risk in Panel B. Both panels provide consistent results since they show that *DEFAULT* and *Illiquidity* are the risk factors that tend to be more closely correlated with the moments of the returns distribution for all analysed horizons. Also both panels indicate that the market risk premium, the detrended risk-free rate, and the employment growth rate display high values for the *R*-squared at longer horizons. As in the case of Table 2, for the same regression, *R*-squared is higher when moments of the distribution are estimated conditionally. Interestingly enough, neither set of moments seems to contain much information on the two Fama-French risk factors.

To further illustrate the two most consistent relationships presented in Table 3, Figure 2 displays the actual values of illiquidity and default risks at the 12-month horizon with respect to their fitted values according to regression (12) using conditional moments as explanatory variables. It is striking the ability of the non-Gaussian determinants of the variance risk premium to explain the overall trend and some of the fluctuations in illiquidity and default risk.

An additional and important issue needs to be addressed. We should provide a more precise analysis on whether it is skewness or kurtosis the more important moment explaining the different risk indicators, as well as the variance risk premium.

The results in Panel A of Table 3 show that coefficients associated to one or both variables related to skewness and kurtosis are statistically significant depending upon the dependent variable and the horizon while, in general, the third explanatory variable in equation (12) seems not to be relevant. However, in order to analyse which cross product (either skewness or kurtosis) is the explanatory variable with more information content, we estimate again a set of regressions based on equations (8) and (12) in which one of the three explanatory variables is excluded. *R*-squared statistics from these estimations are presented in Table 4. The first block in this table refers to the estimation of equation (8) (with the variance risk premium as the dependent variable) while the following blocks refer to the estimation of equation (12) for the four dependent variables with the highest *R*-squared in Table 3. For comparability, the first row in each block also provides the *R*-squared from the estimation of the full regression. Regarding the variance risk premium, we find very similar evidence at the shortest and medium horizons. The cross-product $\sigma_{Wt}(K_{Wt}-1)$ dominates the overall explanatory of these regressions. This is not surprising given the statistical significance of the coefficients associated with the cross-product of kurtosis and volatility shown in Table 2. Things are not so clear for 12-month horizon, where both skewness and kurtosis seem to contain relevant and distinct information on the variance risk premium.

Regarding the rest of dependent variables, with the exception of the market risk premium at the 1-month horizon, the *R*-squared drops substantially when we take $\sigma_{Wt}(K_{Wt}-1)$ out of the regression. Specifically, for the 12-month horizon as an example, the *R*-squared of 0.314 for the market risk factor drops to 0.253 if we take the $\sigma_{Wt}S_{Wt}$ cross-product out of regression (12). It drops to essentially zero if we drop $\sigma_{Wt}(K_{Wt}-1)$ from that regression, and it only falls to 0.301 if we take the ν_{Wt} term out of the regression. In this case, the relevance of the cross product of volatility and kurtosis is largest. For the other risk indicators we have similar evidence: the *R-square* of 0.577 in the full regression for *DEFAULT* risk drops to 0.462, 0.017 and 0.575, respectively, the *R-square* of 0.351 for employment growth drops to 0.310, 0.024 and 0.273, respectively, the *R-square* of 0.350 for the Illiquidity risk factor drops to 0.336, 0.087 and 0.341. The evidence seems to be quite consistent across alternative dependent variables. The cross-product of kurtosis and volatility is a key determinant of the variability of aggregate financial and macroeconomic risks.

5. The Hedging Performance of the Variance Risk Premium against Economic Risk Factors

Up to this point, we have found evidence that the variance risk premium responds to changes in higher order moments of the distribution of market returns, suggesting that the variance swap may offer hedging against time variation and non-Normality in the distribution of returns. But the similar evidence we have found for standard indicators of different types of risk also suggests that we may be able to identify specific types of risk against which variance swaps may offer coverage. In fact, it looks as if the kurtosis term is the more relevant explanatory power both in (8) and (12), which only reinforces the suggestion to directly relate the variance risk premium to the factors for different types of economic and financial risk. To analyze the ability of the variance swap contract to hedge the various types of aggregate risk, we estimate the regressions,

$$vrp_{t,t+\tau} = \alpha + \beta' X_{t,t+\tau} + \varepsilon_{t,t+\tau} \quad (\tau = 1, 2, 3, 6, \text{ and } 12), \qquad (13)$$

where X is a vector of variables representing a specific type of economic or financial risk. The time indexes in (13) reflect the fact that we are looking for the possibility that the variance swap offers advanced coverage for risk that may materialize over the maturity life of the swap contract.

In consistency with the previous section, we consider three sources of risk: equity market risk, interest rate risk, and business cycle risk. The hedging ability of the variance swap against the equity market risk comes from the definition of the contract. The basic intuition behind the variance swap is that investing in volatility appears attractive because volatility shocks are known to be negatively correlated with stock index returns. Thus, adding volatility exposure to an equity portfolio should improve risk diversification. In that sense, we would expect a negative relationship between the variance risk premium and any indicator of stock market risk. Moreover, the volatility of a stock market index increases during recessions, so that a variance swap contract will provide the desired protection if the variance risk premium is higher in anticipation of these stressed periods. For that reason we also analyze the relationship between the variance risk premium and variables representing other types of risk as proxied by interest rates or business cycle indicators. It should be noted that if the variance swap fulfils its role as a hedge against volatility, it will bear a negative relationship with any variable indicating “good news”, and a positive relationship with any indicator of “bad news”.

The first group of variables considers the change in the market index, as the main source of equity risk, but also the size and value risk factors of Fama-French and the innovation in the price-dividend ratio, as additional sources of market risk. We report the estimation results for different maturities, and for the equity risk group, $X = [R_w - R_f, SMB, HML, PD]'$, in Panel A of Table 5. We are interested on the type of risk embedded in the two Fama-French factors and dividend yield that is different from the main source of risk, generated by stock market fluctuations. Hence, we take the residual of a linear projection of each of the three factors on the market index returns as the size, value, and dividend risk component orthogonal to market risk. As with the market index itself, we expect a negative relationship between the variance risk premium and the estimated components of size and value factors, and of the price-dividend ratio that are orthogonal to the market index.

Given our previous evidence, it seems reasonable to expect that the variance swap may also provide protection against interest rate risk. The second group of variables considers three potential sources of risk based on interest rates. First, we take fluctuations in the detrended level of the risk-free real interest rate as the main indicator of interest rate risk, the trend being defined as the average level of the real rate over the last year. Interpreting increases in this variable as bad news, we would expect a positive relationship with the variance risk premium. We also analyze whether the variance risk premium maintains a negative relationship with surprises in the slope of the yield curve and a positive relation with the innovations in the default rate, which could indicate protection against a potential company default. Then $X = [\, R_f STATE,\ TERM,\ DEFAULT\,]'$ and estimation results are presented in Panel B of Table 5. Again, we take the components of *TERM* and *DEFAULT* which are orthogonal to the main source of risk in this group defined by $R_f STATE$.

Finally, we consider the possibility that the variance swap might provide a hedge against negative developments in the business cycle. We use the growth rate of per capita real aggregate non-durable consumption, total employment growth rate, and the market-wide illiquidity surprises as business cycle indicators. In this case, we analyze the relationship between variance risk premium and each one of these three variables individually and the estimation results are reported in the three sections of Panel C of Table 5. We expect a negative relationship between the variance swap premium and the future growth rates of the two macroeconomic indicators, and a positive relation with our measure of aggregate illiquidity shocks.

Before analyzing the results, it bears pointing out that the use of innovations to the risk indicators over the maturity of the swap contract is crucial in our analysis. We are searching for possible evidence that the variance risk premium agreed upon at time t might anticipate future surprises in the different risk indicators between t and $t+\tau$. In general, the correlation will be higher between the variance risk premium and the risk indicator itself, but it might be argued that such correlation is spuriously produced by the persistence in the risk indicators calculated over τ months. To avoid that justified criticism, we correlate the variance risk premium with the innovations or surprises in risk indicators.

Generally speaking, results show widespread evidence in favor of the variance swap playing a significant role as a hedge against a variety of risks. Panel A of Table 5 shows the variance risk premium to be strongly and negatively related to market returns at all maturities. It also shows a negative relationship with changes in the difference between returns to firms with high and low book-to-market ratio, with changes in the difference between market returns to small and large firms, and with the price-dividend ratio. These are the components of the Fama and French (1993) factors and price-dividend ratio that are unrelated to market returns. Hence, the negative estimated coefficients suggest that the variance swap may provide a significant hedge not only against market risk, but also against the specific components of size and value aggregate risks, as well as against shocks to the dividend-price ratio which are not correlated with the market index.[9] A difference between the shorter and the longer maturities is the fact that for the former, variance risk premium seems to relate closely to future developments in the market index and in the price-dividend ratio, with the two Fama-French factors not adding significant information. At the two longest maturities, the situation reverses, and the variance risk premium displays significant correlation with future unexpected changes in the two Fama-French factors, in addition to that contained on the market index. The last row in Panel A displays the *R*-square from a regression that uses the market excess return as the only explanatory factor, showing that the hedge possibilities against risks other than unexpected changes in the index return are significant for the shorter and longest horizons.

Panel B of Table 5 reports the evidence regarding interest rate risk. The difference between the real interest rate over the maturity of the variance swap and the average level of the real rate over the last year acts as a proxy for an interest rate surprise, expecting a positive relationship with the variance risk premium. This does not seem to be the case for any maturity, although the coefficients are estimated with low precision. A flattening of the term structure is known to anticipate a recession, so we

[9] In regressions not reported in this paper, we employ the price-dividend ratio by itself, rather than just its orthogonal component to the market index to check whether the price-dividend is a more appropriate risk factor than the market index itself. It turns out that variance swap premia seem to anticipate future fluctuations in the price-dividend ratio at least as well as fluctuations in the market index.

would expect a potentially negative relationship between the variance risk premium and the innovation in the *TERM* factor. Finally, a positive relationship between the variance risk premium and surprises in the *DEFAULT* factor is expected. It should be recalled that we use the innovations in both *TERM* and *DEFAULT* state variables, after extracting from them the information which is common to fluctuations in interest rates. Over the whole spectrum of maturities considered, the variance risk premium seems to anticipate future fluctuations in *DEFAULT* but not in *TERM*, thereby suggesting that variance swaps may provide a better hedge against default risk than against changes in the slope of the yield curve. The comparison of *R*-squares at the bottom of Panel B of Table 5 shows that the correlation of the variance risk premium with the specific risk component in *DEFAULT* is very significant. [10]

Panel C of Table 5 contains the evidence on business cycle risks. It is interesting to see that the variance risk premium displays a significant negative relationship with the consumption growth rate at all maturities except the shortest one. Hence, long positions on the variance swap contract seem to provide insurance not only with respect to market equity risk, but also to real macroeconomic risks. It might be thought that the correlation we present is spurious, being the consumption growth a proxy for conditions in the stock market or for the level of interest rates. However, an additional analysis, not included in the paper, suggests that this is not the case, since there is correlation between the variance risk premium and consumption growth which is additional to the correlation between the variance risk premium and both, the stock market and the level of interest rates.[11] Similar results are obtained when we use employment growth as an indicator of business cycle risk. The variance risk premium hedges employment risk at the intermediate and longest horizons. Finally, the variance swap seems to also provide hedge against aggregate illiquidity risk. The results show a positive and strongly significant relationship between the variance risk premium and innovations to aggregate illiquidity for all horizons. Interestingly, this positive relationship is maintained if we also add the market return on the regressions, so that market-wide illiquidity seems to be an additional risk factor over and above market risk.

[10] The last row in Panel B of Table 5 reports the *R*-squared statistic from a single regression that considers R_f*STATE* as the only explanatory variable.
[11] This is potentially interesting from the point of view of asset pricing, since any equilibrium model would imply a correlation between the excess return on the swap, captured here by the variance risk premium, and consumption growth.

By and large, the evidence in this section is consistent with that presented in Section 4. It indicates that the variance risk premium is able to anticipate different kinds of risk embedded in traditional state variables. Such risks go beyond the type of risk in stock market returns or in the level of interest rates. In addition to those, we believe that it is especially interesting the consistent correlations we have provided between the variance risk premium and *DEFAULT* risk at the different horizons and as well as with indictors of business cycle risks.

6. Tests of Mean-Variance Spanning with the Variance Swap as the Test Asset

In previous sections we have found evidence suggesting a significant hedging ability in variance swaps against a variety of types of risk. Since this includes market risk, as well as interest rate and macroeconomics risks, our findings suggest that variance swaps may be intrinsically different from standard assets as represented by the stock market index or corporate and government bond yields. If this were the case, variance swaps would not be redundant assets and then they would contribute to improve the investment opportunity set. We test this hypothesis in this section.

To formally test our hypothesis, we conduct a mean-variance spanning test.[12] The idea of this test is simple. A set of K benchmark assets spans a larger set of $N+K$ assets if the two sets of assets share the same minimum variance frontier. Then, the N assets are dominated by the K assets or, equivalently, an investor that has the K assets cannot benefit by investing in the additional set of N assets.

More formally, let $R_t = \left[R_{1t}, R_{2t}\right]'$ be a ($N+K$)-vector of returns on the K benchmark assets (R_{1t}) and on the N test assets (R_{2t}). And let $\mu = E\left(R_t\right)$ and $V = Var\left(R_t\right)$ be the vector of mean returns and the covariance matrix of returns, respectively. The set of the benchmark assets spans the set of the benchmark assets plus the test assets if and only if two restrictions hold:

$$H_0 : \alpha = 0_N, \delta = 0_N \qquad (14)$$

[12] Huberman and Kandel (1987) were the first authors to formalize this issue as a multivariate statistical test, but we follow the implementation in Kan and Zhou (2008).

where $\delta = 1_N - \beta 1_K$, and α is the intercept and β the slope of the projection of R_{2t} on R_{1t}. From the two-fund separation theorem it is possible to interpret the null hypothesis in equation (14) in terms of characteristics of the tangency portfolio and the global minimum variance portfolio on the minimum variance frontier obtained with the $N + K$ assets. Specifically, $\alpha = 0_N$ implies that the tangency portfolio has zero weights in the N assets, and $\delta = 0_N$ implies that the global minimum variance portfolio has zero weights in the N assets.

Jobson and Korkie (1989) rewrite the likelihood ratio test statistic for the null hypothesis of spanning initially proposed by Huberman and Kandel (1987) in order to provide a geometrical interpretation. For the case of a single test asset, the test statistic is,

$$F = \frac{T-K-1}{2}\left[\left(\frac{c}{c_1}\right)\left(\frac{1+{}^{d}\!/_{c}}{1+{}^{d_1}\!/_{c_1}}\right)-1\right] \xrightarrow{d} F_{2,(T-K-1)} \tag{15}$$

where $c = 1'_{N+K}V^{-1}1_{N+K}$, $d = ac - b^2$, $a = \mu' V^{-1}\mu$, $b = 1'_{N+K}V^{-1}\mu$, for the full set of assets (N+K). The corresponding four constants for the set of the K benchmark assets: c_1, d_1, a_1 and b_1 are defined similarly. The first factor inside the squared bracket compares the standard deviation of the global minimum variance portfolios on the two minimum variance frontiers, with K assets and with N+K assets, while the second parenthesis compares the two tangency portfolios on the alternative frontiers.

Finally, it is also possible to compare the two minimum variance frontiers following a step-down procedure.[13] The step-down procedure is a sequential test whose first step consists of testing whether $\alpha = 0_N$, while the test of $\delta = 0_N$ conditional on $\alpha = 0_N$ is conducted in the second step. To test $\alpha = 0_N$ we use the statistic:

$$F_1 = \left(\frac{T-K-N}{N}\right)\left(\frac{a-a_1}{1+a_1}\right) \xrightarrow{d} F_{N,(T-K-N)}. \tag{16}$$

[13] See Anderson (1984) for a general description of this procedure, and Kan and Zhou (2008) for a particular application to an international data set.

And to test $\delta = 0_N$ conditional on $\alpha = 0_N$ we employ the statistic given by,

$$F_2 = \left(\frac{T-K-N+1}{N}\right)\left[\left(\frac{c+d}{c_1+d_1}\right)\left(\frac{1+a_1}{1+a}\right)-1\right] \xrightarrow{d} F_{N,(T-K-N+1)}. \quad (17)$$

We apply the spanning test for the comparison between the minimum variance frontier generated by four assets (the stock market index, the Aaa corporate bond yield, the Baa corporate bond yield, and the 10-year government bond yield) and the minimum variance frontier that is obtained when we add the variance swap. Results regarding both the global and sequential tests are contained in Table 6.

Results from the global test, in the left panel, show that the traditional *F*-test rejects spanning at conventional significance levels for all horizons. Figure 3 offers an illustration of the competing minimum variance frontiers for one- and six-months horizons. In order to analyse the sources of this rejection, we also conduct the sequential test with the first row testing for the restriction that the tangency portfolio has a zero weight in the test asset, and the second row testing for the restriction that the global minimum variance portfolio has a zero weight in the variance swap. We can conclude that while the tangency portfolio can be improved at all horizons, the evidence is particularly strong at the longest horizons. On the contrary, the strong evidence of improving the investment opportunity is associated with the global minimum portfolio rather than with the tangency portfolio at the shortest horizons. In any case, in all types of tests, we systematically reject spanning, suggesting that the variance risk premium contains incremental relevant information not included in the benchmark assets.

7. Conclusions

We have shown that the variance risk premium at different horizons responds to fears by investors to time-varying deviations from Normality in returns. We have also provided evidence that some indicators of default risk, illiquidity risk and business cycle risks are statistically related to the same deviations from Normality. This common influence suggests that the variance swap is a financial instrument that may offer coverage against financial and macroeconomic risks, since they also respond to similar deviations from Normality. Indeed, we show that the excess return on the variance swap contract hedges against equity market risks, interest rate and business cycle risks. We regard as particularly interesting the correlation we have documented between the variance risk premium and future surprises in default and business cycle risk indicators. Since variance swaps offer hedge for risks other than the market, we test for spanning, systematically rejecting the null hypothesis. This suggests the variance risk premium contains incremental relevant information not included in the chosen benchmark assets, which consist of corporate and government bond yields, and the S&P market portfolio return. Hence, the variance swap contract enhances the investment opportunity set available to investors relative to equity and bond yields.

References

Amengual, D. (2009), *The Term Structure of Variance Risk Premia*, Working Paper, Department of Economics, Princeton University.

Amihud, Y. (2002), *Illiquidity and Stock Returns: Cross-Section and Time-Series Effects*, Journal of Financial Markets 5, 31-56.

Anderson, T. W. (1984), *An Introduction of Multivariate Statistical Analysis*. Wiley, New York.

Carr, P. and L. Wu (2009), *Variance Risk Premia*, Review of Financial Studies 22, 1311-1341.

Chabi-Yo, F. (2009), *Pricing Kernels with Coskewness and Volatility Risk*, Working Paper, Fisher College of Business, Ohio State University.

Driessen, J., P. Maenhout, and G. Vilkov (2009), *The Price of Correlation Risk: Evidence from Equity Options*, Journal of Finance 64, 3, 1377-1406.

Egloff, D., M. Leippold, and L. Wu (2007), *Variance Risk Dynamics, Variance Risk Premia, and Optimal Variance Swap Investments*, Forthcoming in the Journal of Financial and Quantitative Analysis.

Fama, E., and K. French (1993), *Common Risk Factors in the Returns on Stocks and Bonds*, Journal of Financial Economics 33, 3-56.

Gallant, A. and G. Tauchen (1989), *Semi Non-parametric Estimation of Conditionally Constrained Heterogeneou Processes: Asset Pricing Applications*, Econometrica 57, 1091-1120.

Hasbrouck, J. (2009), *Trading Costs and Returns for US Equities: Estimating Effective Costs from Daily Data*, Journal of Finance, 64, 3, 1445-1477.

Harvey, C. and A. Siddique (2000), *Conditional Skewness in Asset Pricing Tests*, Journal of Finance 55, 1263-1295.

Huberman, G. and S. Kandel (1987), *Mean-variance Spanning*, Journal of Finance, 42, 873-888.

Jobson, J. D. and B. Korkie (1989), *A Performance Interpretation of Multivariate Tests of Asset Set Intersections, Spanning and Mean-variance Efficiency*, Journal of Financial and Quantitative Analysis, 24, 185-204.

Kan, R., and G. Zhou (2008), *Tests of Mean-Variance Spanning*, Working Paper, Olin Business School, Washington University, St. Louis.

Kraus, A., and R. Litzenberger (1976), *Skewness Preference and the Valuation of Risky Assets*, Journal of Finance 31, 1085-1100.

León, A., G. Serna, and G. Rubio (2005), *Autoregressive Conditional Volatility, Skewness and Kurtosis*, Quarterly Review of Economics and Finance 42, 599-618.

Márquez, E., B. Nieto, and G. Rubio (2009), *Consumption, Liquidity, and the Cross-Sectional Variation of Expected Returns*, VI Research Workshop on Asset Pricing, IE Business School.

Rubinstein, M. (1973), *The Fundamental Theorem of Parameter-Preference Security Valuation*, Journal of Financial and Quantitative Analysis 8, 61-69.

Todorov, V. (2009), *Variance Risk Premia Dynamics: The Role of Jumps*, Review of Financial Studies, 23, 1, 345-383.

Vilkov, G. (2008), *Variance Risk Premium Demystified*, Working Paper, INSEAD.

Table 1
Variance Risk Premia: Descriptive Statistics

Panel A: Descriptive Statistics					
	VRP1	*VRP2*	*VRP3*	*VRP6*	*VRP12*
Mean	-0.646	-0.635	-0.659	-0.694	-0.736
Median	-0.697	-0.682	-0.719	-0.751	-0.734
Maximun	0.834	0.952	0.841	0.706	0.441
Minimum	-1.556	-1.612	-1.631	-1.576	-1.600
Panel B: Linear Correlations					
	VRP1	*VRP2*	*VRP3*	*VRP6*	*VRP12*
VRP1	1	0.793	0.659	0.402	0.224
VRP2		1	0.910	0.650	0.453
VRP3			1	0.798	0.574
VRP6				1	0.793
VRP12					1

VRP is the variance risk premium associated with the alternative horizons of the variance swap contract going from 1 to 12 months. It is computed as the difference between the ex-post realized variance at the end of the swap contract and the observed variance swap rate

Table 2
The Sources of the Variance Risk Premium

Panel A: Unconditional Moments						
	τ =1 month		*τ = 6 months*		*τ =12 months*	
Constant	-0.110	(0.021)	-0.018	(0.074)	-0.046	(0.082)
λ_W	0.208	(0.414)	-1.144	(1.437)	-4.197	(1.527)
λ_{SKD}	-0.142	(0.063)	-0.231	(0.107)	-0.230	(0.146)
λ_{VOL}	-0.005	(0.016)	-0.107	(0.080)	-0.106	(0.086)
R^2	0.074		0.157		0.180	
Panel B: Conditional Moments						
	τ =1 month		*τ = 6 months*		*τ =12 months*	
R^2	0.303		0.222		0.297	

The table reports results from the estimation of the following regression

$$VRP_{t,t+\tau} = \lambda_0 + \lambda_W \left(\sigma_{Wt,t+\tau} S_{Wt,t+\tau}\right) + \lambda_{SKD} \left(\sigma_{Wt,t+\tau} (K_{Wt,t+\tau} - 1)\right) + \lambda_{VOL} v_{Wt,t+\tau} + \varepsilon_{t,t+\tau}, \ \tau = 1,6,12$$

where $VRP_{t,t+\tau}$ is the Variance Risk Premium computed as the difference between the ex-post realized variance at the end of the swap contract ($t+\tau$) and the observed variance swap rate. σ_W, S_W, and K_W represent the standard deviation, the skewness and the kurtosis of the market return, respectively, and $v_{Wt} = Cov_t\left(\sigma^2_{Wt+1}, R^2_{Wt+1}\right) / Var_t\left(\sigma^2_{Wt+1}\right)$. In Panel A, all three moments are estimated with intra-daily data within the period corresponding to the swap maturity (1 month, 6 months or 12 months). Each row in this panel reports the estimates and their corresponding standard error in parentheses. The last row is the *R*-squared of the regression. In Panel B, the three moments are estimated using a GARCH framework from equations (8)-(10). In this case, the *R*-squared of each regression is the only reported statistic.

Table 3
The Relationship between the Moments of the Returns Distribution and State Variables

Panel A: Unconditional Moments							
		τ =1 month		*τ = 6 months*		*τ =12 months*	
$R_W - R_f$	*Constant*	0.866	(0.471)	2.042	(0.465)	1.755	(0.421)
	λ_W	46.27	(10.12)	-17.37	(9.276)	-21.36	(11.75)
	λ_{SKD}	-1.189	(1.206)	-2.982	(0.757)	-3.440	(0.862)
	λ_{VOL}	0.077	(0.536)	-0.357	(0.596)	0.457	(0.443)
	R^2	0.192		0.210		0.314	
SMB	*Constant*	0.623	(0.666)	0.41	(0.457)	0.736	(0.387)
	λ_W	18.11	(10.29)	10.32	(8.827)	10.30	(8.515)
	λ_{SKD}	0.682	(0.996)	0.304	(0.692)	0.353	(0.691)
	λ_{VOL}	-0.841	(0.609)	-0.43	(0.503)	-0.829	(0.357)
	R^2	0.054		0.040		0.121	
HML	*Constant*	1.031	(0.429)	0.252	(0.491)	-0.100	(0.492)
	λ_W	-26.56	(8.484)	5.191	(18.76)	4.893	(15.87)
	λ_{SKD}	-2.319	(1.055)	0.265	(1.378)	0.823	(1.097)
	λ_{VOL}	0.380	(0.393)	0.148	(0.336)	0.175	(0.333)
	R^2	0.134		0.005		0.026	
PD	*Constant*	1.348	(0.483)	7.734	(2.951)	7.363	(5.471)
	λ_W	22.42	(7.001)	-84.74	(49.17)	-192.5	(127.6)
	λ_{SKD}	-2.330	(0.911)	-9.954	(4.449)	-17.92	(10.21)
	λ_{VOL}	-0.289	(0.300)	-1.021	(3.244)	8.169	(5.292)
	R^2	0.103		0.084		0.117	
R_f *STATE*	*Constant*	0.011	(0.033)	0.031	(0.026)	0.035	(0.018)
	λ_W	-0.048	(0.423)	-0.438	(0.434)	-0.063	(0.435)
	λ_{SKD}	-0.035	(0.059)	-0.098	(0.044)	-0.146	(0.033)
	λ_{VOL}	-0.004	(0.014)	0.010	(0.021)	0.039	(0.014)
	R^2	0.005		0.104		0.393	
TERM	*Constant*	0.011	(0.007)	0.002	(0.025)	-0.019	(0.039)
	λ_W	0.029	(0.080)	-0.285	(0.467)	-0.520	(0.761)
	λ_{SKD}	-0.018	(0.013)	0.060	(0.037)	0.167	(0.052)
	λ_{VOL}	-0.007	(0.004)	-0.042	(0.022)	-0.085	(0.032)
	R^2	0.04		0.121		0.275	
DEFAULT	*Constant*	-0.004	(0.001)	-0.026	(0.007)	-0.043	(0.008)
	λ_W	-0.056	(0.025)	0.312	(0.151)	0.755	(0.247)
	λ_{SKD}	0.008	(0.003)	0.061	(0.011)	0.118	(0.013)
	λ_{VOL}	0.001	(0.001)	0.002	(0.006)	-0.005	(0.008)
	R^2	0.109		0.354		0.577	
Consumption Growth	*Constant*	0.205	(0.031)	0.193	(0.024)	0.172	(0.025)
	λ_W	0.782	(0.690)	-0.403	(0.486)	0.361	(0.551)
	λ_{SKD}	-0.169	(0.089)	-0.088	(0.039)	-0.053	(0.045)
	λ_{VOL}	0.025	(0.022)	0.019	(0.024)	0.030	(0.026)
	R^2	0.05		0.079		0.083	
Employment Growth	*Constant*	0.115	(0.022)	0.148	(0.033)	0.168	(0.032)
	λ_W	-0.010	(0.257)	-0.809	(0.664)	-1.313	(0.924)
	λ_{SKD}	-0.076	(0.028)	-0.207	(0.049)	-0.261	(0.064)
	λ_{VOL}	0.030	(0.018)	0.070	(0.032)	0.083	(0.029)
	R^2	0.055		0.268		0.351	

		τ =1 month		τ = 6 months		τ =12 months	
Agg. Illiq. Shocks	*Constant*	-5.945	(2.116)	-24.94	(7.772)	-44.61	(10.12)
	λ_W	-4.638	(36.24)	168.0	(166.8)	218.0	(231.0)
	λ_{SKD}	15.01	(3.849)	44.90	(12.41)	74.33	(13.57)
	λ_{VOL}	1.201	(1.571)	5.440	(8.405)	8.228	(9.065)
	R^2	0.092		0.212		0.350	

Panel B: Conditional Moments

	τ =1 month	τ = 6 months	τ =12 months
$R_W - R_f$	0.031	0.202	0.185
SMB	0.026	0.206	0.099
HML	0.025	0.079	0.077
PD	0.050	0.133	0.064
R_f *STATE*	0.007	0.104	0.185
TERM	0.015	0.053	0.041
DEFAULT	0.207	0.517	0.579
Consumption Growth	0.014	0.118	0.274
Employment Growth	0.014	0.139	0.244
Agg. Illiq. Shocks	0.238	0.427	0.517

The table reports results from the estimation of the following regression

$$Y_{t,t+\tau} = \lambda_0 + \lambda_W \left(\sigma_{Wt,t+\tau} S_{Wt,t+\tau}\right) + \lambda_{SKD} \left(\sigma_{Wt,t+\tau} (K_{Wt,t+\tau} - 1)\right) + \lambda_{VOL} v_{Wt,t+\tau} + \varepsilon_{t,t+\tau}\,,\ \tau = 1,6,12$$

The dependent variable (Y) changes for each row as indicated in the first column of the table: the excess market return ($R_W - R_f$), the size premium (*SMB*), the value premium (*HML*), the price-dividend ratio (*PD*), the relative risk free rate (R_f *STATE*), the slope of the yield curve (*TERM*), a default premium (*DEFAULT*) computed as the difference between *Baa* corporate bonds and government bonds, the aggregate consumption growth rate, the employment growth rate, and an aggregate measure of the illiquidity shocks. σ_W , S_W , and K_W represent the standard deviation, skewness and kurtosis of the market return, respectively, and $v_{Wt} = Cov_t\left(\sigma^2_{Wt+1}, R^2_{Wt+1}\right) / Var_t\left(\sigma^2_{Wt+1}\right)$. In Panel A, all three moments are estimated with intra-daily data within the period corresponding to the swap maturity (1 month, 6 months or 12 months). Each row in this panel reports the estimates and their corresponding standard error in parenthesis. The last row is the *R*-squared of the regression. In Panel B, the three moments are estimated using a GARCH framework from equations (8)-(10). In this case, the *R*-squared of each regression is the only reported statistic.

Table 4
Contribution of each Moment of the Return Distribution to the Explanation of the Variance Risk Premium and some State Variables

		τ =1 month	τ = 6 months	τ =12 months
VRP	$\lambda_W, \lambda_{SKD}, \lambda_{VOL}$	0.074	0.157	0.180
	$\lambda_{SKD}, \lambda_{VOL}$	0.071	0.144	0.062
	λ_W, λ_{VOL}	0.004	0.081	0.110
	λ_W, λ_{SKD}	0.073	0.113	0.145
$R_W - R_f$	$\lambda_W, \lambda_{SKD}, \lambda_{VOL}$	0.192	0.210	0.314
	$\lambda_{SKD}, \lambda_{VOL}$	0.004	0.170	0.253
	λ_W, λ_{VOL}	0.185	0.035	0.000
	λ_W, λ_{SKD}	0.191	0.203	0.301
DEFAULT	$\lambda_W, \lambda_{SKD}, \lambda_{VOL}$	0.109	0.354	0.577
	$\lambda_{SKD}, \lambda_{VOL}$	0.055	0.299	0.462
	λ_W, λ_{VOL}	0.051	0.039	0.017
	λ_W, λ_{SKD}	0.103	0.352	0.575
Employment Growth	$\lambda_W, \lambda_{SKD}, \lambda_{VOL}$	0.055	0.268	0.351
	$\lambda_{SKD}, \lambda_{VOL}$	0.055	0.245	0.310
	λ_W, λ_{VOL}	0.024	0.043	0.024
	λ_W, λ_{SKD}	0.023	0.197	0.273
Agg. Illiq. Shocks	$\lambda_W, \lambda_{SKD}, \lambda_{VOL}$	0.092	0.212	0.350
	$\lambda_{SKD}, \lambda_{VOL}$	0.092	0.195	0.336
	λ_W, λ_{VOL}	0.011	0.056	0.087
	λ_W, λ_{SKD}	0.089	0.205	0.341

The table reports *R*-squared statistics from the estimation of the following regressions

$$VRP_{t,t+\tau} = \lambda_0 + \lambda_W \left(\sigma_{Wt,t+\tau} S_{Wt,t+\tau}\right) + \lambda_{SKD} \left(\sigma_{Wt,t+\tau}(K_{Wt,t+\tau} - 1)\right) + \lambda_{VOL} v_{Wt,t+\tau} + \varepsilon_{t,t+\tau}$$

$$Y_{t,t+\tau} = \lambda_0 + \lambda_W \left(\sigma_{Wt,t+\tau} S_{Wt,t+\tau}\right) + \lambda_{SKD} \left(\sigma_{Wt,t+\tau}(K_{Wt,t+\tau} - 1)\right) + \lambda_{VOL} v_{Wt,t+\tau} + \varepsilon_{t,t+\tau}, \ \tau = 1,6,12$$

The dependent variable (Y), indicated in the first column of the table, is the variance risk premium (*VRP*), the excess market return ($R_W - R_f$), a default premium (*DEFAULT*), computed as the difference between *Baa* corporate bonds and government bonds, the employment growth rate, and an aggregate measure of the illiquidity shocks. For each group of results, the first row reports the *R*-squared of the full equation (considering the three explanatory variables). The following three rows report the *R*-squared of a regression including two out of the three explanatory variables, which are indicated in the second column of the table. As before, σ_W, S_W, and K_W represent the standard deviation, skewness and kurtosis of the market return, respectively, and $v_{Wt} = Cov_t\left(\sigma^2_{Wt+1}, R^2_{Wt+1}\right) / Var_t\left(\sigma^2_{Wt+1}\right)$. All moments have been estimated with intra-daily data within the period corresponding to the swap maturity.

Table 5
The Hedging Ability of the Variance Swap Contract

Panel A: Equity Risks					
	τ =1 month	*τ = 2 months*	*τ = 3 months*	*τ = 6 months*	*τ = 12 months*
$R_W - R_f$	-4.601 (0.693)	-8.388 (0.881)	-11.918 (1.105)	-16.619 (1.654)	-14.496 (2.285)
*SMB**	-1.227 (0.876)	-1.486 (1.144)	-2.377 (1.405)	-3.271 (2.160)	-8.901 (3.366)
*HML**	-2.084 (1.156)	-2.245 (1.345)	-2.829 (1.565)	-4.576 (1.955)	-6.218 (2.831)
*PD**	-3.774 (1.167)	-2.582 (0.885)	-1.787 (0.835)	-0.390 (0.609)	0.656 (0.434)
Adj. R^2	0.316	0.440	0.488	0.456	0.316
Adj. R^2 (R_W)	0.226	0.388	0.461	0.436	0.216
Panel B: Interest Rate Risks					
	τ =1 month	*τ = 2 months*	*τ = 3 months*	*τ = 6 months*	*τ = 12 months*
R_f *STATE*	-7.847 (19.158)	-19.606 (24.392)	-38.780 (30.352)	-80.000 (38.780)	-244.260 (53.830)
*TERM **	-21.730 (91.193)	-22.376 (75.001)	-33.299 (72.335)	31.790 (53.665)	3.179 (51.890)
*DEFAULT **	1617.06 (368.39)	1186.73 (223.26)	1151.29 (178.07)	1096.42 (121.51)	575.440 (106.31)
Adj. R^2	0.131	0.193	0.272	0.423	0.311
Adj. R^2 (R_f)	-0.007	-0.004	0.002	0.024	0.151
Panel C: Business Cycle Risks					
	τ =1 month	*τ = 2 months*	*τ = 3 months*	*τ = 6 months*	*τ = 12 months*
Consumption Growth	-11.39 (13.73)	-75.95 (23.92)	-142.15 (31.98)	-280.11 (42.00)	-301.63 (52.08)
Adj. R^2	-0.002	0.065	0.125	0.249	0.199
Employment Growth	-37.46 (28.16)	-54.67 (32.47)	-89.87 (34.58)	-119.61 (35.06)	-140.84 (34.63)
Adj. R^2	0.006	0.027	0.043	0.078	0.115
Agg. Illiq. Shocks	0.961 (0.228)	0.995 (0.165)	0.972 (0.153)	0.935 (0.143)	0.629 (0.126)
Adj. R^2	0.125	0.232	0.254	0.271	0.184

This table reports the slope coefficients, autocorrelation-robust standard errors in parentheses, and *R*-squared coefficients from the $vrp_{t,t+\tau} = \alpha + \beta' X_{t,t+\tau} + \varepsilon_{t,t+\tau}$, where $vrp_{t,t+\tau}$ is the log return on holding a variance swap with maturity in $t+\tau$. In Panel A, equity risk is analyzed by considering four variables included in vector *X*: innovations in the excess market return $\left(R_W - R_f\right)$, the size premium (*SMB*), the value premium (*HML*), and the price-dividend ratio (*PD*). In Panel B, we analyze the relationship between the variance risk premium and three variables representing interest rates risk: innovations in the relative risk free rate (R_f *STATE*), the slope of the yield curve (*TERM*) and a default premium (*DEFAULT*). All variables marked with * indicate that we take the residuals relative to the main source of risk: market return in Panel A and the risk free rate in Panel B. The second *Adj.* R^2 line refers to the regression that includes only the main source of risk as explanatory variable. Panel C reports the business cycle risk coefficients corresponding to simple OLS regressions with consumption growth, employment growth, and an illiquidity shocks measure, respectively, as the only independent variables.

Table 6
Tests of Mean-Variance Spanning with the Variance Swap as the Test Asset

	τ =1 month	*τ = 2 months*	*τ = 3 months*	*τ = 6 months*	*τ = 12 months*
Global Test:					
$H_0 : \alpha = 0, \delta = 0$	22.300 (0.000)	21.653 (0.000)	20.785 (0.000)	21.978 (0.000)	34.808 (0.000)
Step Down Test:					
1) $H_0 : \alpha = 0$	11.871 (0.001)	13.529 (0.000)	15.942 (0.000)	29.145 (0.000)	60.766 (0.000)
2) $H_0 : \delta = 0 \,/\, \alpha = 0$	30.167 (0.000)	27.123 (0.000)	22.950 (0.000)	12.141 (0.001)	6.033 (0.015)

These spanning tests compare the minimum variance frontier of four assets (the stock market index, the Aaa corporate bond index, the Baa corporate bond index, and the 10-year government bond) with the minimum variance frontier of five assets (the four previous assets plus the variance swap). From the minimum variance frontier of the five assets, $\alpha = 0$ is a restriction that implies that the tangency portfolio on the minimum variance frontier has a zero weight in the test asset (the variance swap) and $\delta = 0$ is a restriction that implies that the global minimum variance portfolio has a zero weight in the test asset. Then, the first column reports the results for the global test while second column reports results from the two steps of a step down test. For all cases, the first number is the test statistic and the number in parentheses is the *p-value* from its finite sample distribution.

Figure 1
Variance Swap Rate and Realized Variance for Different Maturities

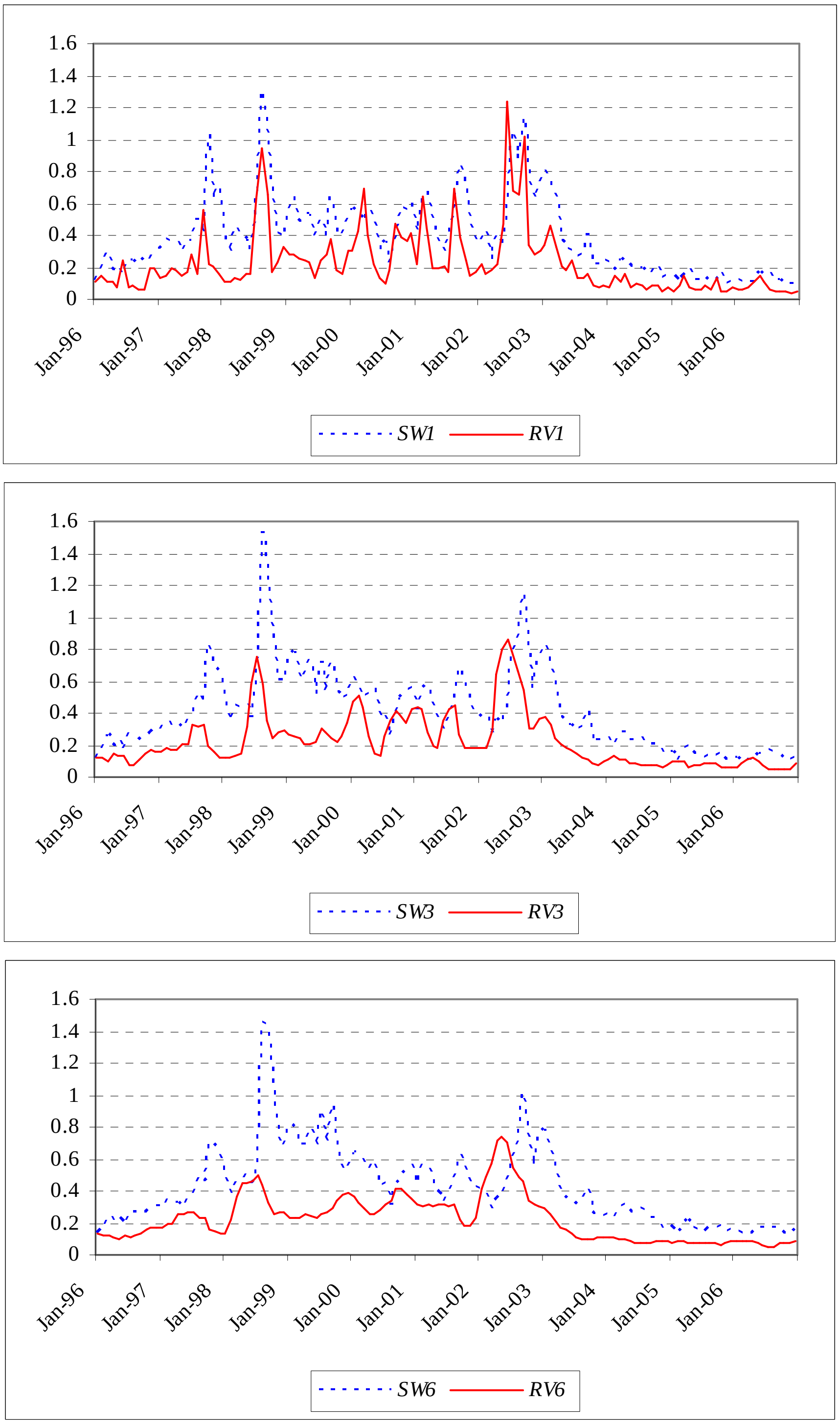
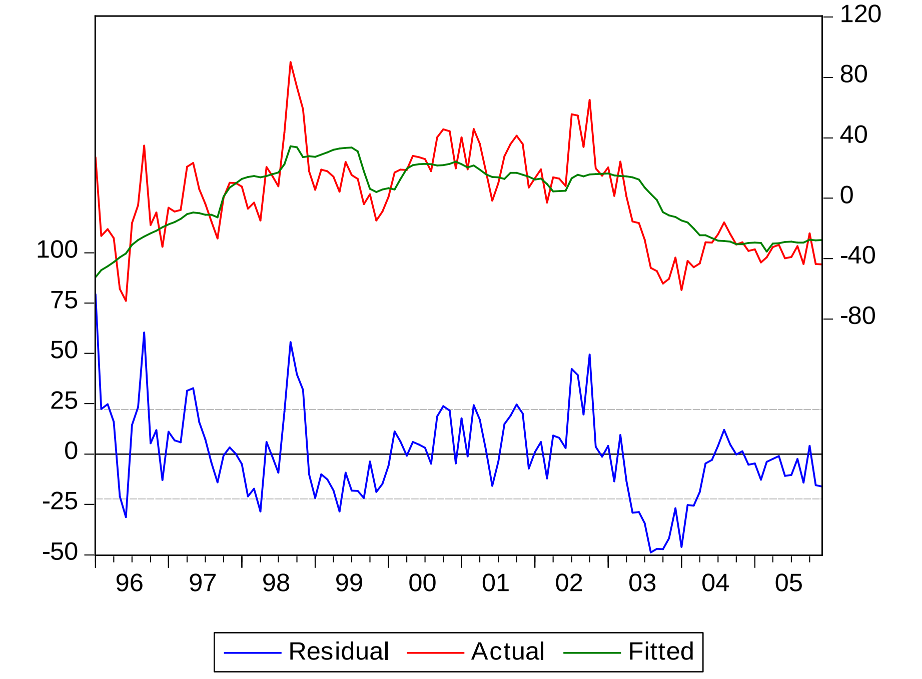


Figure 2
Actual vs. Fitted Values of Illiquidity and Default Risks against Non-Normal Determinants of the Variance Risk Premium at the 12-month Horizon

Illiquidity Risk



Default Risk

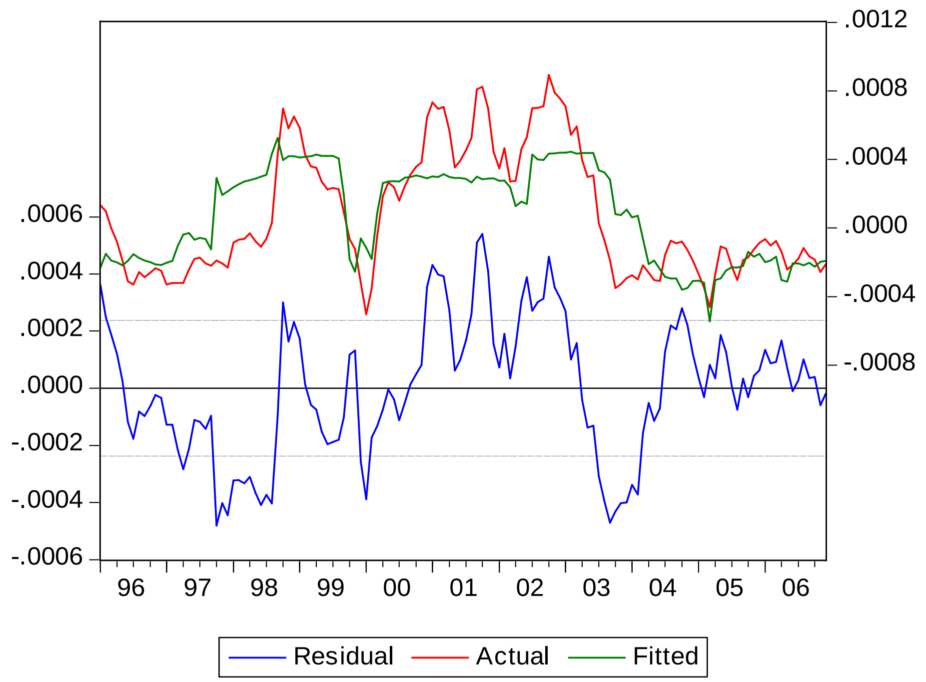


Figure 3
Minimum Variance Frontiers with and without the Variance Swap