ϕ^4 theory in $1+d$ dimensions at high temperature: Dimensional reduction

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The ϕ^4 theory in $1+d$ dimensions is analyzed at high temperatures in the imaginary-time formalism. General results are given for the leading high-temperature contributions to all renormalized Green's functions. The latter are generated by a high-temperature partition function which describes another ϕ^4 theory in d spatial dimensions with special mass renormalizations. The triviality/nontriviality of the $(\phi^4)_{1+3}$ theory is discussed briefly.

I. INTRODUCTION

It has been suggested that quantum field theories could become simpler (dimensionally reduced) at high temperatures,[1] which would be a variant of the Appelquist-Carazzone theorem.[2] Recently, that simplification for quantum electrodynamics in two spatial dimensions has been established to all perturbative orders.[3] It would be interesting to provide detailed proofs of high-temperature dimensional reduction for other theories. The main purpose of this work will be to establish it for the ϕ^4 theory in $1+d$ dimensions. The results are stated in Secs. II and IV, while the proofs are outlined in Sec. III. We shall also discuss in Sec. IV the triviality/nontriviality of the $(\phi^4)_{1+3}$ theory.

II. GENERAL FRAMEWORK AND HIGH-TEMPERATURE RESULTS FOR $(\phi^4)_{1+3}$ THEORY

First, we consider a quantized real scalar Bose (renormalized) field ϕ, in $d=3$ spatial dimensions with quartic coupling and renormalized mass m (≥ 0) and coupling constant λ in thermodynamical equilibrium at finite temperature β^{-1}. In the imaginary-time $(\tau, -\beta/2 \leq \tau \leq +\beta/2)$ formalism,[4] the partition function including an external source $J(\tau\mathbf{x})[J(-\frac{1}{2}\beta\mathbf{x})=J(\frac{1}{2}\beta\mathbf{x})]$ reads

$$Z[J]=N\int[D\phi]\exp\left[-S+\int_{-\beta/2}^{+\beta/2}d\tau\int d^3\mathbf{x}\,J(\tau\mathbf{x})\phi(\tau\mathbf{x})\right]$$

$$=N\exp\left[\sum_{n=2}^{+\infty}\frac{1}{n!}\int\left[\prod_{h=1}^{n}d\tau_h d^3\mathbf{x}_h J(\tau_h\mathbf{x}_h)\right]\Delta_n(\tau_1\mathbf{x}_1\cdots\tau_n\mathbf{x}_n)\right], \tag{2.1}$$

$$S=\int_{-\beta/2}^{+\beta/2}d\tau\int d^3\mathbf{x}\left\{\frac{Z_\phi}{2}\left[\left(\frac{\partial\phi}{\partial\tau}\right)^2+(\nabla\phi)^2\right]+\tfrac{1}{2}(m^2+\delta m^2)\phi^2+\frac{\lambda}{4!}Z_\lambda\phi^4\right\}. \tag{2.2}$$

N is a normalization constant, ∇ is the spatial gradient, and

$$\phi=\phi(\tau\mathbf{x}),\quad [D\phi]=\prod_{\tau,\mathbf{x}}d\phi(\tau\mathbf{x}),\quad \phi\left(-\frac{\beta}{2}\mathbf{x}\right)=\phi\left(\frac{\beta}{2}\mathbf{x}\right).$$

The finite-temperature renormalized connected Green's functions are

$$\Delta_n(\tau_1\mathbf{x}_1\cdots\tau_n\mathbf{x}_n)=\beta^{-n+1}\sum_{r_1=-\infty}^{+\infty}\cdots\sum_{r_{n-1}=-\infty}^{+\infty}\int\left[\prod_{h=1}^{n-1}\frac{d^3\mathbf{K}_h}{(2\pi)^3}\right]\tilde{\Delta}_n(K_1\cdots K_n)$$

$$\times\exp\left[i\sum_{j=1}^{n-1}[\mathbf{K}_j\cdot(\mathbf{x}_j-\mathbf{x}_n)-\omega(r_j)(\tau_j-\tau_n)]\right],\quad \sum_{j=1}^{n}K_j=0, \tag{2.3}$$

$$K_j=(i\omega(r_j),\mathbf{K}_j),\quad \omega(r_j)=\beta^{-1}2\pi r_j,\quad r_j=0,\pm1,\pm2,\ldots. \tag{2.4}$$

δm^2, Z_ϕ, and Z_λ are the standard mass renormalization counterterm and the renormalization constants, respectively, associated with the field strength and the coupling constant, *all at zero temperature.*[5] They suffice to render all $\tilde{\Delta}_n$'s ultraviolet convergent at any finite temperature in perturbation theory.[6]

Next, we consider the following partition function $Z[j]_{\rm HT}$ [depending on a τ-*independent* real scalar field $\phi(\mathbf{x})$ and external source $j(\mathbf{x})$] and connected Green's functions $\Delta_n(\mathbf{x}_1\cdots\mathbf{x}_n)_{\rm HT}$ in three spatial dimensions $(m\geq 0)$:

$$Z[j]_{\rm HT}=N\int[d\phi]\exp\left[-\beta S_{\rm HT}+\beta\int d^3\mathbf{x}\,j(\mathbf{x})\phi(\mathbf{x})\right]$$

$$=N\exp\left[\sum_{n=2}^{+\infty}\frac{1}{n!}\int\left[\prod_{h=1}^{n}d^3\mathbf{x}_h j(\mathbf{x}_h)\right]\beta^{n/2}\Delta_n(\mathbf{x}_1\cdots\mathbf{x}_n)_{\rm HT}\right], \tag{2.5}$$

$$S_{\rm HT}=\int d^3\mathbf{x}\left[\tfrac{1}{2}[\nabla\phi(\mathbf{x})]^2+\tfrac{1}{2}(m^2+\delta m_{\rm HT}{}^2)\phi(\mathbf{x})^2+\frac{\lambda}{4!}\phi(\mathbf{x})^4\right], \tag{2.6}$$

$$\Delta_n(\mathbf{x}_1\cdots\mathbf{x}_n)_{\rm HT}=\int\left[\prod_{h=1}^{n-1}\frac{d^3\mathbf{K}_h}{(2\pi)^3}\right]\tilde{\Delta}_n(\mathbf{K}_1\cdots\mathbf{K}_n)_{\rm HT}\exp\left[i\sum_{j=1}^{n-1}\mathbf{K}_j\cdot(\mathbf{x}_j-\mathbf{x}_n)\right],\quad \sum_{j=1}^{n}\mathbf{K}_j=0\ , \tag{2.7}$$

$[d\phi]=\prod_x d\phi(\mathbf{x})$, while m and λ are the *same zero-temperature* renormalized quantities as in (2.2). The subscript HT means "high temperature." We suppose that $[\nu(\mathbf{k})=(m^2+\mathbf{k}^2)^{1/2}]$

$$\delta m_{\rm HT}{}^2=\delta m_{\rm HT,1}{}^2({\rm a})-\delta m_{\rm HT,2}{}^2({\rm a})-\delta m_{\rm HT,1}{}^2({\rm b})+\delta m_{\rm HT,2}{}^2({\rm b})\ , \tag{2.8}$$

$$\delta m_{{\rm HT},j}{}^2({\rm a})=\frac{\lambda}{2}\int\frac{d^3\mathbf{K}}{(2\pi)^3\nu(\mathbf{K})}\times\begin{cases}\{\exp[\beta\nu(\mathbf{K})]-1\}^{-1}, & j=1\ ,\\ [\beta\nu(\mathbf{K})]^{-1}, & j=2\ ,\end{cases} \tag{2.9}$$

$$\delta m_{{\rm HT},j}{}^2({\rm b})=\lambda^2\int\frac{d^3\mathbf{K}_1 d^3\mathbf{K}_2}{(2\pi)^6}\times\begin{cases}F(\mathbf{K}_1,\mathbf{K}_2;0), & j=1\ ,\\ \beta^{-2}[\nu(\mathbf{K}_1)\nu(\mathbf{K}_2)\nu(\mathbf{K}_1+\mathbf{K}_2)]^{-2}, & j=2\ ,\end{cases} \tag{2.10}$$

$$F(\mathbf{K}_1,\mathbf{K}_2;\mathbf{K})=\lim_{\epsilon\to0^+}\left[\prod_{j=1}^{2}\left[\int_{-i\infty+\epsilon}^{+i\infty+\epsilon}\frac{d\omega_j}{2\pi i}[\exp(\beta\omega_j)-1]^{-1}+\int_{-i\infty-\epsilon}^{+i\infty-\epsilon}\frac{d\omega_j}{2\pi i}[\exp(-\beta\omega_j)-1]^{-1}\right]\right.$$
$$\left.\times[\omega_j{}^2-\nu(\mathbf{K}_j)^2]^{-1}\right][(\omega_1+\omega_2)^2-\nu(\mathbf{K}_1+\mathbf{K}_2-\mathbf{K})^2]^{-1}\ . \tag{2.11}$$

The origin of $\delta m_{\rm HT}{}^2$ lies in the four-dimensional theory (2.1) and (2.2) while $\delta m_{{\rm HT},j}{}^2$(a) and $\delta m_{{\rm HT},j}{}^2$(b) are, respectively, associated to the Feynman diagrams in Figs. 1(a) and 1(b) (see Sec. III). Notice that $\delta m_{\rm HT,1}{}^2$(a),$\delta m_{\rm HT,1}{}^2$(b) are ultraviolet convergent while $\delta m_{\rm HT,2}{}^2$(a),$\delta m_{\rm HT,2}{}^2$(b) are divergent. For given β, $Z[j]_{\rm HT}$ describes a superrenormalizable $(\phi^4)_3$ theory. In fact, in $d=3$, the only Feynman diagrams which are ultraviolet divergent are the ones given in Figs. 1(a) and 1(b) and those which contain the latter as subdiagrams. Those ultraviolet divergences are canceled, respectively, by $\delta m_{\rm HT,2}{}^2$(a) and $\delta m_{\rm HT,2}{}^2$(b). Thus, all $\tilde{\Delta}_n(\mathbf{K}_1\cdots\mathbf{K}_n)_{\rm HT}$'s are ultraviolet finite.

Finally, we go to high temperatures, where $\beta m \ll 1$ and, eventually, $\beta\to0$. Then, our main results, to all orders of perturbation theory, are the following. For fixed

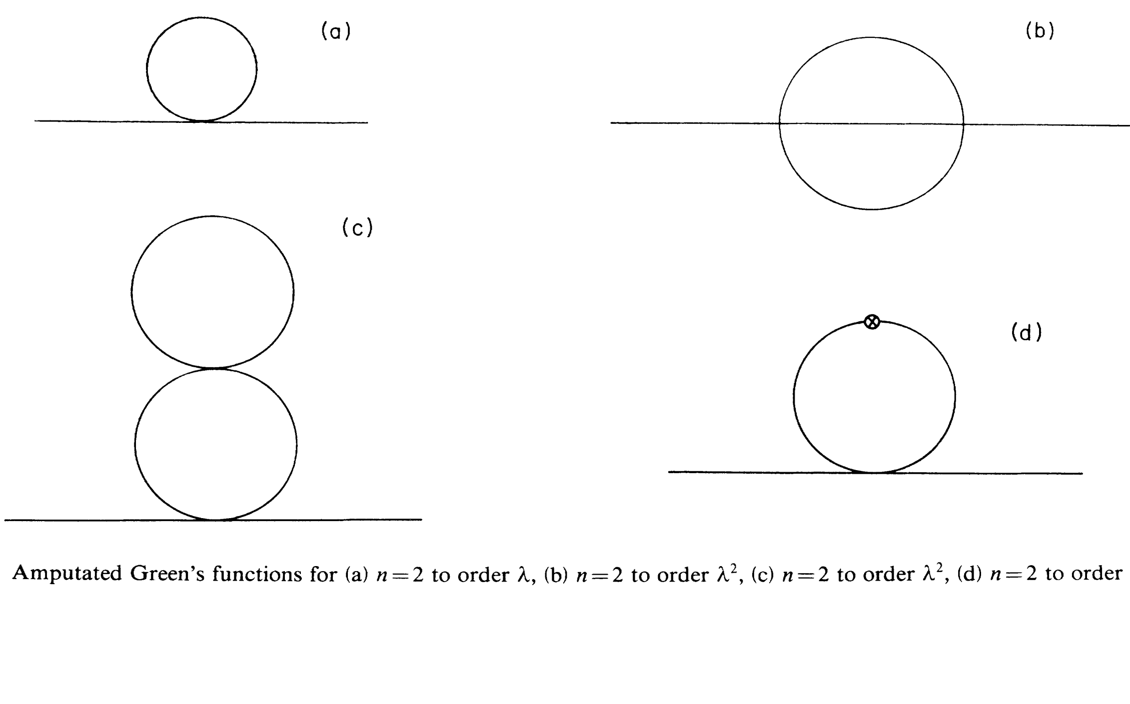

FIG. 1. Amputated Green's functions for (a) $n=2$ to order λ, (b) $n=2$ to order λ^2, (c) $n=2$ to order λ^2, (d) $n=2$ to order λ^2.

$\mathbf{K}_1 \cdots \mathbf{K}_{n-1}(\beta|\mathbf{K}_j| \ll 1,\ j=1,\ldots,n-1)$ and vanishing external frequencies

$$\tilde{\Delta}_n((0,\mathbf{K}_1)\cdots(0,\mathbf{K}_n)) \underset{\beta\to 0}{\sim} \beta^{n/2-1}\tilde{\Delta}_n(\mathbf{K}_1\cdots\mathbf{K}_n)_{\mathrm{HT}} \tag{2.12}$$

due to the specific renormalizations (2.8)–(2.11). Let

$$J(\tau\mathbf{x}) \underset{\beta\to 0}{\to} j(\mathbf{x})\ , \tag{2.13}$$

where

$$(2\pi)^{-3/2}\int d^3\mathbf{x}\, j(\mathbf{x})\exp(i\mathbf{K}\cdot\mathbf{x})$$

takes on its largest values for $|\mathbf{K}| \ll \beta^{-1}$ and is entirely negligible for $|\mathbf{K}| \sim \beta^{-1}$ and $|\mathbf{K}| \gg \beta^{-1}$, by assumption. Then

$$Z[J] \underset{\beta\to 0}{\sim} Z[j]_{\mathrm{HT}}\ . \tag{2.14}$$

III. PROOF OF DIMENSIONAL REDUCTION [(2.12) AND (2.14)] FOR $m>0$

(1) Recall that for any function $f(\omega)$ which is analytic in a neighborhood of the imaginary axis $[\omega(n)=\beta^{-1}2\pi n,\ n=0,\pm1,\pm2,\ldots,\epsilon\to 0^+]$ (Ref. 6)

$$\frac{1}{\beta}\sum_{n=-\infty}^{+\infty} f(\omega=i\omega(n)) = \int_{-i\infty}^{+i\infty}\frac{d\omega}{2\pi i}f(\omega) + \int_{-i\infty+\epsilon}^{+i\infty+\epsilon}\frac{d\omega}{2\pi i}\frac{f(\omega)}{\exp(\beta\omega)-1} + \int_{-i\infty-\epsilon}^{+i\infty-\epsilon}\frac{d\omega}{2\pi i}\frac{f(\omega)}{\exp(-\beta\omega)-1}\ . \tag{3.1}$$

(2) We shall deal with the renormalized perturbative contributions to the *amputated* connected Green's functions $\tilde{\Delta}_n^{\mathrm{amp}}$ which are obtained from those for $\tilde{\Delta}_n$ by factoring out all propagators for external lines. We omit details regarding Feynman rules, diagrammatics, etc.[4,5]

(3) The contribution to the renormalized $\tilde{\Delta}_2(K)^{\mathrm{amp}}$ from Fig. 1(a) is

$$\tilde{\Delta}_2^{(a)}(K)^{\mathrm{amp}} = \frac{\lambda}{2}\beta^{-1}\sum_{n=-\infty}^{+\infty}\int\frac{d^3\mathbf{q}}{(2\pi)^3}[\omega(n)^2+\nu(\mathbf{q})^2]^{-1} + (\delta m^2)^{(1)}\ , \tag{3.2}$$

$(\delta m^2)^{(1)}$ being the (negative ultraviolet-divergent) term[5] of order λ from δm^2 in (2.2). By using (3.1) in (3.5), canceling $(\delta m^2)^{(1)}$ with the contribution arising from the first term on the right-hand side (RHS) of (3.1) one gets the convergent result

$$\tilde{\Delta}_2^{(a)}(K)^{\mathrm{amp}} = \frac{\lambda}{2}\int\frac{d^3\mathbf{q}}{(2\pi)^3\nu(\mathbf{q})}\{\exp[\beta\nu(\mathbf{q})]-1\}^{-1}\ . \tag{3.3}$$

For $K=(0,\mathbf{K})$, (3.3) coincides exactly with the net contribution to $\tilde{\Delta}_2(\mathbf{K})_{\mathrm{HT}}^{\mathrm{amp}}$ arising from $Z[j]_{\mathrm{HT}}$ and corresponding to (i) Fig. 1(a) in $d=3$, and (ii) $\delta m_{\mathrm{HT},1}{}^2(\mathrm{a}) - \delta m_{\mathrm{HT},2}{}^2(\mathrm{a})$.

(4) We now turn to the contributions for the amputated $\tilde{\Delta}_2(K)$ for $K=(0,\mathbf{K})$ associated to (i) Figs. 1(b) and 1(c), (ii) the terms of orders λ and λ^2 in δm^2 and the λ^2 terms in λZ_λ (that is, $\lambda Z_\lambda^{(1)}$) at zero temperature,[5] and (iii) the λ term in Z_ϕ at zero temperature.[5] We use (3.1), cancel all ultraviolet divergences, and concentrate on the leading terms as $\beta\to 0$ for fixed $\mathbf{K}$. In the contribution associated to Fig. 1(c), we replace $[\exp(\pm\beta\omega)-1]^{-1}$ by $\pm(\beta\omega)^{-1}$ in the frequency integral associated to the lower loop (as all integrations related to it have now become ultraviolet convergent). Thus, we get

$$\tilde{\Delta}_2^{(b,c)}((0,\mathbf{K}))^{\mathrm{amp}} \underset{\beta\to 0}{\sim} -A(\mathbf{K}) - \delta m_{\mathrm{HT},1}{}^2(\mathrm{a})\frac{\lambda}{\beta}\times\int\frac{d^3\mathbf{q}}{(2\pi)^3}[\nu(\mathbf{q})]^{-4}\ , \tag{3.4}$$

$$A(\mathbf{K}) = \lambda^2\int\frac{d^3\mathbf{K}_1 d^3\mathbf{K}_2}{(2\pi)^6}F(\mathbf{K}_1,\mathbf{K}_2;\mathbf{K}) = \delta m_{\mathrm{HT},1}{}^2(\mathrm{b}) + A(\mathbf{K}) - A(\mathbf{K}=0)\ . \tag{3.5}$$

The first and second terms on the RHS of (3.4) are ultraviolet convergent and come from Figs. 1(b) and 1(c), respectively. $\delta m_{\mathrm{HT},1}{}^2(\mathrm{a})$ and $F(\mathbf{K}_1,\mathbf{K}_2;\mathbf{K})$ are given in Eqs. (2.9) and (2.11). In Eq. (3.5) we have used $A(\mathbf{K}=0)=\delta m_{\mathrm{HT},1}{}^2(\mathrm{b})$. If $\beta\to 0$ (for fixed $\mathbf{K}$), we replace $[\exp(\pm\beta\omega_j)-1]^{-1}$ by $\pm(\beta\omega_j)^{-1}$ inside $A(\mathbf{K})-A(\mathbf{K}=0)$, since the logarithmic ultraviolet divergence is removed by the subtraction. Then,

$$A(\mathbf{K}) - A(\mathbf{K}=0) \underset{\beta\to 0}{\sim} \frac{\lambda^2}{\beta^2}\int\frac{d^3\mathbf{K}_1 d^3\mathbf{K}_2}{(2\pi)^6}[\nu(\mathbf{K}_1)\nu(\mathbf{K}_2)]^{-2}\{[\nu(\mathbf{K}_1+\mathbf{K}_2-\mathbf{K})]^{-2} - [\nu(\mathbf{K}_1+\mathbf{K}_2)]^{-2}\}\ . \tag{3.6}$$

Now, let us consider the overall amputated contribution $\tilde{\Delta}_2^{(b,c)}(\mathbf{K})_{\mathrm{HT}}^{\mathrm{amp}}$ to $\tilde{\Delta}_2(\mathbf{K})_{\mathrm{HT}}$ generated by $Z[j]_{\mathrm{HT}}$ and associated to (i) Figs. 1(b) and 1(c) in $d=3$, both of which provide negative divergent contributions [that from Fig. 1(b) equals the first term on the RHS of (3.6)], (ii) $-\delta m_{\mathrm{HT},1}{}^2(\mathrm{b}) + \delta m_{\mathrm{HT},2}{}^2(\mathrm{b})$ which are of order λ^2, (iii) the λ^2 diagram in Fig. 1(d) in $d=3$ (which carries an overall negative sign), where, in turn, the small circle with a cross corresponds to $\delta m_{\mathrm{HT},1}{}^2(\mathrm{a}) - \delta m_{\mathrm{HT},2}{}^2(\mathrm{a})$. We cancel the actual (negative) divergent contribution from Fig. 1(c) in $d=3$ with the positive (also divergent) one from the $-\delta m_{\mathrm{HT},2}{}^2(\mathrm{a})$ term in Fig. 1(d). Then, the resulting ultraviolet finite expression for $\tilde{\Delta}_2^{(b,c)}(\mathbf{K})_{\mathrm{HT}}^{\mathrm{amp}}$ coincides with $\tilde{\Delta}_2^{(b,c)}((0,\mathbf{K}))^{\mathrm{amp}}$, as given by Eqs. (3.4)–(3.6).

(5) The analysis for other one-particle-irreducible graphs and for diagrams which contain the ones studied above as subdiagrams can be carried out similarly. In all cases, one verifies the validity of (2.12).

(6) We now outline the proof of (2.14) under the stated assumptions on J. Using (2.13), (2.3), and integrating over τ's in the right-most side of (2.1), one is led to restrict to vanishing external frequencies for all connected Green's functions. The dominant contributions to the three-momenta integrations for the latter in (2.1) come from three-momenta much smaller than β^{-1} due to the properties assumed for $j(\mathbf{x})$. By using (2.12), one gets (2.14) directly.

IV. DISCUSSION AND REMARKS

(1) Let $\beta \to 0$ for given $m>0$ as in Sec. III and, later, set $m=0$. If $\lambda>0$, the dimensionally reduced $(\phi^4)_3$ theory in (2.5)–(2.11) with $m=0$ is infrared finite. This follows by including $\delta m_{\mathrm{HT},1}{}^2$(a) (which is positive and finite) in the bare propagator for (2.5) and (2.6), which becomes $[\mathbf{K}^2+\delta m_{\mathrm{HT},1}{}^2(\mathrm{a})]^{-1}$, and noticing that $-\delta m_{\mathrm{HT},2}{}^2$(a) and $-\delta m_{\mathrm{HT},1}{}^2$(b) + $\delta m_{\mathrm{HT},2}{}^2$(b) are infrared finite. Alternatively, set $m=0$ first, consider the Green's functions for $(\phi^4)_{1+3}$ theory at finite temperature for nonexceptional momenta (no partial sum of external momenta vanishes) in order to avoid infrared divergences and take $\beta \to 0$: the leading contributions to all Green's functions are generated by (2.5)–(2.11) with $m=0$ and are infrared finite as stated above (even for exceptional momenta).

(2) We shall give without proof the analogues for $d=1,2$ spatial dimensions of (2.12) and (2.14) (with $d^3\mathbf{x}$'s and $d^3\mathbf{K}$'s replaced by $d^d\mathbf{x}$'s, $d^d\mathbf{K}$'s) for $m>0$. Equations (2.1)–(2.4) also hold, with the same meaning, provided that (i) $Z_\lambda = Z_\phi = 1$, and (ii) δm^2 be the standard zero-temperature mass renormalization counterterms associated to Figs. 1(a) and 1(b) for $d=2$ [1(a) for $d=1$]. The new partition function $Z[j]_{\mathrm{HT}}$ with an external source j and connected Green's functions $\Delta_{n\mathrm{HT}}$ are given through Eqs. (2.5)–(2.7) with $\delta m_{\mathrm{HT}}{}^2 = [\delta m_{\mathrm{HT},1}{}^2(\mathrm{a}) - \delta m_{\mathrm{HT},2}{}^2(\mathrm{a})]\delta_{2,d}$ ($\delta_{2,d}$ being a Kronecker delta) and Eq. (2.9). Then, (2.12) and (2.14) hold for $d=1,2$. If $m=0$ infrared divergences appear.

(3) The triviality/nontriviality of the zero-temperature $(\phi^4)_{1+3}$ theory has attracted much interest.[7,8] Would triviality ($\lambda=0$) be true at zero temperature, it would also hold at a small nonzero one but perhaps not for all temperatures. It has been argued that the zero-temperature $(\phi^4)_{1+3}$ theory is nontrivial with either (i) a nonzero mass and an infinitesimally negative *bare* coupling constant (λ_0) (Ref. 9) or (ii) zero mass and infinitesimally positive λ_0 (Ref. 10). Unfortunately, it is not easy to restate those conditions in terms of the *renormalized* coupling constant λ. Notice that if $\lambda<0$, the reduced theory (2.5)–(2.11) would have negative noninfinitesimal coupling λ/β and its stability would be in question. Let the zero-temperature $(\phi^4)_{1+3}$ theory with $\lambda>0$ (even if it would be infinitesimal) be nontrivial, possibly in the massless case (which could escape to the analyses which have arrived at triviality). Then, the reduced theory (2.5)–(2.11) (which would be infrared finite if $m=0$, as commented above) would have noninfinitesimal coupling λ/β. Since the strict nontriviality of the massive $(\phi^4)_3$ theory for small positive coupling has been proved[8] and it would also hold, most naturally, for a nonsmall one, one would conclude that for $\lambda>0$ the initial (possibly, massless) $(\phi^4)_{1+3}$ theory would have fallen at high temperature into a class of nontrivial $(\phi^4)_3$ theories. Recall that Alles and Tarrach[11] obtained nontriviality for the $(\phi^4)_{1+3}$ theory at finite temperature (however, see Bardeen and Moshe[12]).

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