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A 3-year longitudinal study of children's comprehension of counting: Do they recognize the optional nature of nonessential counting features?☆

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ABSTRACT

This 3-year longitudinal study examines developmental changes in children's ability to differentiate essential from nonessential counting features. Kindergarteners watched a computer-presented detection task which included three kinds of counts: correct conventional, erroneous and pseudoerrors (with and without statements of cardinal values for the sets). Children had to judge the correctness of those counts and justify their responses. Our data showed that children's explanations provided additional information and thus increased reliability of the assessment. Children were better at detecting erroneous counts than pseudoerrors and at detecting pseudoerrors with cardinal value than pseudoerrors without it. Group analysis showed that children's performance improved with age but analysis of individual differences qualified this result by identifying individual differences in developmental patterns. This study thus provides a more detailed picture of the developmental trajectories of children's comprehension of essential and nonessential counting aspects.

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Understanding how children develop the ability to count is crucial not only from a psychological perspective but also because of its educational importance. After decades of research, recent studies suggest that it is necessary to consider children's comprehension of the essential and nonessential aspects involved in counting (Briars & Siegler, 1984; Escudero, 2012; Kamawar et al., 2010; Laupa & Becker, 2004; LeFevre et al., 2006; Rodríguez, Lago, Enesco, & Guerrero, 2013).

Essential aspects, also known as *logical*[1] *rules*, are indispensable for counting correctly and are defined by the five principles described by Gelman and Gallistel (1978): (a) one-to-one correspondence: assigning a unique tag to each element; (b) stable order: the tags used must be unique, and their order must be the same in successive counts; (c) cardinality: the final tag used has a special significance, as it provides a cardinal value to the set; (d) abstraction: the three previous principles may be applied to any collection of objects; and (e) order irrelevance: the order in which elements are counted does not affect the cardinal value of the set. The first three principles are known as *how-to-count principles* because they define the conceptual structure of counting, and the last two principles are known as *permissibility principles* because they expand and give flexibility to the range of conditions under which the first three may be applied.

Nonessential aspects, or *conventional rules*, depend on the context or common practices. Nonessential aspects regularly appear in the procedures witnessed by children although they are not necessary for correct counting. For example, in Western culture, a common practice consists of counting rows of objects from left to right consecutively, but violating this conventional rule does not lead to incorrect answers as long as the counting principles are correctly applied.

Although Gelman and Gallistel (1978) stated that many counting behaviors are arbitrary, Briars and Siegler (1984) were the first to define four nonessential characteristics: (a) pointing to each object once, (b) starting from one end, (c) counting from left to right, and (d) counting all objects consecutively (adjacency). Recently, Rodríguez et al. (2013) also identified two types of adjacency using explanations offered by children: spatial, which coincides with the type described by Briars and Siegler (1984), and temporal, which involves emitting all of the numerical tags without skipping forwards or backwards, pausing, or iterating.

Children's comprehension of the essential and nonessential aspects of counting has been studied using detection tasks in which the child must observe how a character (e.g., an adult, doll, or puppet) counts and then judge whether the counting has been performed correctly or incorrectly. In some cases, the character counts correctly according to the usual mode (correct conventional counts). In other cases, the character fails to comply with a specific logical rule (erroneous counts), for example by assigning the same numerical tag to two different elements. Finally, at other times, the character counts correctly in a nonconventional mode (nonconventional counts or pseudoerrors), for example beginning to count at an element located in the middle of the row.

Because of inconsistent results from the pioneering studies of Gelman and Meck (1983), Gelman and Meck (1986) and Briars and Siegler (1984), in which the results of the former study revealed the capacity of 3–5-year-olds to correctly detect pseudoerrors or nonconventional counts, while the latter findings revealed the poor performance of children of the same age with similar counts, recent research has expanded the age range to include primary school children (Escudero, 2012; Kamawar et al., 2010; LeFevre et al., 2006; Rodríguez et al., 2013). These investigations corroborate the proposal by Briars and Siegler (1984), which states that the differentiation of the essential and nonessential aspects of counting is not complete at 5 years of age. In general, the findings also agree in demonstrating that children judge both erroneous counts (transgressing logical rules) and unusual counts or pseudoerrors (transgressing conventional rules) as incorrect.

Finally, several studies have shown that children with and without learning disabilities have difficulties in distinguishing between logical and conventional counting rules (Geary, Bow-Thomas, & Yao, 1992; Geary, Hamson, & Hoard, 2000; Geary, Hoard, Byrd-Craven, & DeSoto, 2004). For example, Geary et al. (2004) found that it is necessary for children without learning disabilities to reach 8–9

[1] The term "logical" refers to the conceptual knowledge involved in the counting principles, not to the Piagetian notion of logical development.

years of age before they can attain an 80% success rate when faced with pseudoerrors, and an age of 10–11 years is required for the same result in children with learning disabilities.

Nevertheless, it has recently been demonstrated that certain characteristics of the experimental conditions may render the optional nature of conventional rules more evident to children. In particular, Rodríguez et al. (2013) observed that, if pseudoerrors appeared along with an explicit statement of the cardinal value of the set, the probability that children would accept the pseudoerrors as valid forms of counting increased considerably (42%, vs. 28% correct answers for pseudoerrors with and without cardinal values, respectively, in 5–8-year-olds). According to the authors, the presence of the cardinal value contributed to children's focusing more on the result and thus relativizing the importance of the procedure used to determine it.

Similarly, given that the majority of the studies only asked children to state whether the character has counted correctly or not, several authors recommend further investigation into the factors influencing children in deciding whether a certain type of counting is correct (Kamawar et al., 2010). Although Gelman and Meck (1983), Gelman and Meck (1986) considered spontaneous explanations for children's right or wrong responses, and Geary et al. (2004) questioned children about the validity of one of the pseudoerrors that had earlier been judged as incorrect, researchers did not systematically ask children to justify their answers until the study by Rodríguez et al. (2013). It is the analysis of these justifications that has facilitated not only the avoidance of false positives but also the establishment of conventional rules underlying incorrect answers.

From an educational perspective, development of the comprehension of the logical and conventional rules of counting may be addressed by means of two equally important approaches: (a) identifying general age-related developmental patterns, and (b) focusing on individual differences. Regarding the first approach, the developmental course of children's knowledge of conventional and logical rules is different. In general, logical rules are assumed to show linear improvement with age (Sarnecka & Carey, 2008 for a detailed analysis of children's early comprehension of cardinality). With regard to conventional rules, results from cross-sectional studies have demonstrated distinct developmental patterns. Authors such as Briars and Siegler (1984); Saxe, Becker, Sadeghpour, and Sicilian (1989); and Rodríguez et al. (2013) have determined that children's awareness that the conventional rules are arbitrary characteristics of counting improves with age. However, LeFevre et al. (2006) and Kamawar et al. (2010) observed a curvilinear pattern such that children in the early years of primary education (6–8 years) are more intolerant of deviations from standard procedures than kindergarteners (5–6 years) and older children (10–11 years). Finally, in one of the few investigations evaluating the detection of errors and pseudoerrors using a longitudinal design, Geary and collaborators (2000) found that the performance of participants did not vary from first to second graders.

As for the second approach, which focuses on individual differences in the comprehension of logical and conventional rules, data are still scarce, despite the fact that this information would have a remarkable impact on the design of more efficient educational strategies, as has been the case in other areas of mathematical knowledge. Notable examples are the investigations of Dowker (2005, 2008) and Jordan, Mulhern, and Wylie (2009) regarding students' arithmetic knowledge. Their results suggest that no uniform developmental pattern exists; rather, there are different trajectories based on the skill being studied (e.g., positional value, calculation, and resolution of algorithms) and the initial knowledge of the participants. Recently, Hallett, Nunes, Bryant, and Thorpe (2012) also described important individual differences in children's comprehension of fractions.

Prior research has revealed that children more easily detect noncompliance with logical rules in erroneous counts than they detect the validity of transgressing conventional rules in pseudoerrors. Nevertheless, certain questions remain. First, there are conflicting data about developmental patterns in the comprehension of the essential and nonessential aspects of counting, and no studies of individual differences exist. Moreover, most of the research is cross-sectional, with hardly any longitudinal data. Second, few studies have thoroughly investigated children's responses during the detection task using additional questions. Third, because it is a recent and interesting finding for which little data exist, presentation of the cardinal value in pseudoerrors should again be examined for any positive effects in improving performance.

In the present study, a three-year longitudinal study was conducted with a group of 5–6-year-olds, which coincides with the transition from kindergarten to primary school. Knowledge of counting was evaluated using a detection task involving erroneous trials, conventional correct trials, and pseudo-errors. The objective was to study the development of the ability to discriminate between the logical and conventional rules of counting and to identify any individual differences in this development.

To assess whether the performance level attributed to each participant varied according to the coding criterion adopted, we relied on a semi-structured interview that asked children to justify their responses. In this way, the coding criterion could be based either exclusively on the correct identification of trials (right/wrong answers) or on both identification and justification.

Finally, pseudoerrors were presented with and without cardinal values, and the erroneous counts did not follow the logical and conventional rules. The goal was to establish whether certain conditions clarify the conception of conventional rules as nonessential. More precisely, we aimed to determine whether children's performance improved for pseudoerrors with cardinal value compared with pseudoerrors without cardinal value, as found by Rodríguez et al. (2013), and whether children rejected erroneous counts because the character had transgressed the logical rules without alluding to conventional transgressions. Once more, the relevance of the semi-structured interview is apparent for achieving these objectives.

1. Method

1.1. Participants

Twenty-four children were selected from kindergarten (12 boys). Participants had no learning difficulties or school adjustment problems, and all were from the same center in Madrid. One additional boy was excluded because he left the school prior to completion.

Children's ages on the first measurement occasion (while enrolled in kindergarten) ranged between 63 and 74 months ($M = 68.42$ months, $SD = 3.36$). On the second measurement occasion, during first grade, children were 75–86 months old ($M = 80.42$ months). Finally, on the third measurement occasion, children were in the second grade and 87–98 months old. ($M = 92.42$ months).

Information provided by the school indicated that participants were of middle- and low-middle-income socioeconomic levels.

1.2. Materials and procedures

To conduct the detection task, we used software titled "The Little House of Numbers",[2] which is a modified version of the software used by Rodríguez et al. (2013). This program is a computer game, the format of which added the important components of being both motivational and realistic. The game contained a variety of animated characters: Rosa, the teacher, and Mara, Eva, Tina, and Eli, the children.

To prevent fatigue and possible lack of attention, three semi-structured 15-min interviews were conducted during each of the yearly measurement occasions. The time interval between sessions was 2–3 weeks.

Each session began with Rosa, the teacher character, explaining, "Now we are going to play counting things with a friend."[3] Then, the character entered and introduced herself: "Hello, I am Mara/Eva/Tina/Eli." Rosa continued, "She is learning to count, and so we have to help her, okay? I am going to put some things on the table for her to count." At this moment, a blue curtain lowered from the ceiling and hid Rosa and the table. A few seconds later, the curtain rose, and the objects to be counted appeared. Next, the character began to move and count the row of objects, one item per second, while she reached her arm down to touch each object as she spoke the corresponding tag. To help children identify the counted items, each item swayed slightly when the character put her finger on it. Once the count was performed, Rosa asked, "Has she done it right or has she done it wrong?" After the child answered, the researcher asked questions to explore or clarify the justifications offered,

[2] Registered Intellectual Property Number: M-002197/2012.
[3] Instructions are translated into English as tasks were administered in Spanish.

before proceeding to the next trial: Why?; Why can't we do that? [only applicable when the participant judged the count as incorrect]; When we are counting, can we . . . [the character's counting]?; Has . . . [the character] made or not any other mistake during her counting?

The software allowed trials to be repeated whenever necessary, for example, when the participant did not remember the character's counting. Only 1.91% of the trials were repeated (2.08% of trials in the first measurement occasion, 2.34% in the second, and 1.30% in the third). Justifications were collected by an audio recorder for subsequent transcription and analysis.

In total, 16 trials occurred during the test: four correct conventional counts, four erroneous counts, and eight pseudoerrors, of which four were accompanied by an explicit statement of the cardinal value of the set (i.e., "There are ___ [number of objects]."). The order of presentation of trials was randomized for each session and was constant across participants. In the first session, the order of appearance for the trials was as follows: pseudoerror without cardinal value, conventional, erroneous, pseudoerror with cardinal value, pseudoerror without cardinal value, conventional, erroneous, and pseudoerror with cardinal value. In the second and third sessions, the order was as follows: pseudoerror without cardinal value, conventional, erroneous, and pseudoerror with cardinal value.

The same instructions were given for all trials, and the objects were always presented in a row. The size of the sets varied between 7 and 14 items. Prior investigation demonstrated that the size of the set does not affect children's performance (Kamawar et al., 2010); thus, nonperceptual quantities were used that were still within a range of "easy" counting for participants. Whenever possible, to reduce the demands of the task, the departure from logical and conventional rules occurred in the middle or the end of the row and affected several items.

The correct conventional counts followed the logical rules as well as the conventional rules. The character counted correctly according to the standard procedure: consecutively, from left to right, pointing to each item and saying the numbers aloud in ascending order. These trials were included as a control to guarantee that children understood the instructions and could distinguish the different counts.

Pseudoerrors only transgressed conventional rules. The pseudoerrors coincided with the ones used by Rodríguez et al. (2013) and covered a wide range of conventional rules. The eight trials differed from one another to avoid possible learning effects. In four of the trials, the character explicitly stated the cardinal value of the set (i.e., "There are ___ [number of objects].") after counting.

As for pseudoerrors without cardinal values, in *Pseudoerror 1*, the character skipped one item (the seventh) and counted it at the end. In *Pseudoerror 2*, the character pointed to and tagged the same item three times ("6–6–6"). In *Pseudoerror 3*, the character pretended to have forgotten the number 6 (saying, "1, 2, 3, 4, 5, hmmm") and proceeded to the number 7. Immediately thereafter, she remembered the forgotten number and reversed the direction of counting to say "6!" and then finished tagging and pointing to the remaining items ("8, 9"). In *Pseudoerror 4*, the character pointed to all of the items but only said aloud the even numbers (i.e., counting by twos: "2, 4, 6, 8"). Table 1 lists the conventional rules that were not met in each trial.

Regarding pseudoerrors with cardinal values (Table 1), in *Pseudoerror 5*, the three final items in the row were silently counted, after which the character stated the cardinal value of the set ("There are 10"). In *Pseudoerror 6*, a row composed of soccer balls and basketballs was shown, and the character first pointed to and tagged the soccer balls and then the basketballs but still arrived at the correct cardinal number ("There are 12"). In *Pseudoerror 7*, seven teddy bears appeared in different colors and were each sliced in half. The character pointed to each one of the teddy bear parts and assigned them only one numerical tag, ending up with the correct cardinal number of the set ("There are 7"). Finally, in *Pseudoerror 8*, the character counted all of the elements while switching directions (one on the left and another on the right) but still obtained the correct cardinal value ("There are 8").

The erroneous counts did not meet the logical and conventional rules. In *Error 1*, the character counted backwards from 9 to 3 and incorrectly stated "3" as the cardinal value for the set. In *Error 2*, the row of objects was heterogeneous, composed of sharp and blunt pencils. The error was made when the character counted each type of item separately (from 1 to 4 the sharp pencils and from 1 to 9 the blunt pencils) and stated the cardinal value to be the larger of the subsets. In *Error 3*, the character began to count at the fourth item in the row ("1"), continued from the first on the left ("2"), and arriving again at the fourth object, counted it again ("5"). The character then continued counting

Table 1
Erroneous counts and pseudoerrors: unfulfilled rules.

	N^a	Description	Unfulfilled conventional rules
Error 1	7	Using the mechanically learned rule of the last number word to state the cardinality of the set (transgression of the cardinality principle)	Stating the counting sequence in ascending order
Error 2	13	Counting members of a heterogeneous group as two different subgroups (transgression of the abstraction principle)	Starting from one end Spatial adjacency Left-to-right direction
Error 3	7	Doubly counted item (transgression of the one-to-one correspondence principle)	Starting from one end Spatial adjacency Left-to-right direction
Error 4	12	Assigning different cardinal values to the same set based on the first counted item (transgression of the order irrelevance principle)	Starting from one end
Pseudoerror 1	9	False omission error	Spatial adjacency Left-to-right direction
Pseudoerror 2	10	False repetition error	Spatial and temporal adjacency
Pseudoerror 3	9	False tagging error	Spatial and temporal adjacency Left-to-right direction
Pseudoerror 4	8	False numeral omission error	Counting out loud
Pseudoerror 5	10	Silent counting of the last three items (correct cardinal value)	Counting out loud Pointing to or touching objects
Pseudoerror 6	12	Counting alternate items (correct cardinal value)	Spatial adjacency Left-to-right direction
Pseudoerror 7	7	Changing the counting unit (correct cardinal value)	Each discrete element as an item Spatial adjacency Left-to-right direction
Pseudoerror 8	8	Counting in alternate directions (correct cardinal value)	Spatial adjacency Left-to-right direction

[a] *N* refers to the size of the set.

until reaching the end of the row. In *Error 4*, the character assigned two different cardinal values to the set depending on where she began to count (saying, "There are 12," if she began with the first object and saying, "There are 4," if she began with the ninth object) (see Table 1).

Taking into account the stricter criterion (correct identification and justification), participants' responses to the erroneous counts were coded as correct if they said the counts were incorrect and justified their judgment by referencing the logical rule that was breached. In order for the responses to the pseudoerrors to be coded as correct, the children were required to accept the pseudoerrors as valid forms of counting and to argue that the logical rules had been respected. Finally, for the conventional counts, all of the responses were considered adequate if they were identified as valid forms of counting.

Inter-coder agreement, calculated using 18% of the total set of trials, was 95%. Any discrepancies were reviewed and debated until agreement was reached.

2. Results

The criterion for a correct response (correct identification and justification) was stricter than that used in most prior investigations (correct identification only). Thus, we used a *t*-test for related samples to assess for differences in scores depending on the criterion adopted. This test revealed significant differences at measurement occasions 1, $t(23) = 6.04$, $p < .001$, and 2, $t(23) = 3.31$, $p < .01$, but not occasion 3, $t(23) = 1.48$, $p = .153$. Children scored higher using the correct identification criterion than the correct identification and justification criterion at measurement occasions 1 ($M = 5.21$, $SD = 1.7$ and $M = 3.71$, $SD = 2.18$, respectively) and 2 ($M = 5.96$, $SD = 2.24$ and $M = 5.08$, $SD = 2.72$, respectively), but not occasion 3 ($M = 6.67$, $SD = 2.81$ and $M = 6.29$, $SD = 3.56$, respectively).

These discrepancies cannot be attributed to the children having difficulties justifying their responses because all participants appropriately justified their responses in every one of the trials on all occasions. However, justifications were not always correct. For example, for the erroneous counts, some children stated that the character had counted wrong (correct identification) but

Table 2
Means and standard deviations (in parentheses) of the correct responses for the detection tasks across measurement occasions.

Detection Task	Measurement 1 (5–6 y.o.) *M* (*SD*)	Measurement 2 (6–7 y.o.) *M* (*SD*)	Measurement 3 (7–8 y.o.) *M* (*SD*)
Erroneous Counts	2.08 (0.93)	2.83 (0.96)	3.17 (1.01)
Pseudoerrors without Cardinal Value	0.88 (1.04)	0.71 (1.04)	1.13 (1.54)
Pseudoerrors with Cardinal Value	0.75 (1.03)	1.54 (1.53)	2.00 (1.67)

Note. Maximum possible score is 4.

argued incorrectly that she had not met conventional rules instead of logical ones. Irene (aged 66 months) said, to justify her answer to *Error 3*, "Because she counted "1" here [while tagging the fourth book in the row] and here, [while tagging the first book in the row], she counted "2" . . . Because we can't count the numbers "1" and "2" in there. We have to count like this: "1" here [while tagging the farthest left book in the row], "2" here, "3" here . . . [pointing to the adjacent items consecutively]." This type of inadequate justification was rare at measurement occasion 3, most likely because the children's knowledge of essential and nonessential aspects of counting had improved. This would also explain the lack of significant differences between both scoring criteria at measurement occasion 3.

Cronbach's alpha coefficient values were $\alpha = .78$ for the "correct identification" criterion and $\alpha = .85$ for "correct identification and justification." As expected, reliability increased when justifications were considered because this additional information allowed not only for demonstrating whether the participants realized the rules that had been transgressed (i.e., identifying false positives or negatives) but also for assessing the importance that children ascribed to the rules.

Turning now to task type, the correct conventional counts were excluded from this analysis because these trials were used as a control for children's comprehension of instructions as well as ability to discriminate between different types of counting. As expected, success rates were extremely high (92% on measurement occasion 1 and 100% on measurement occasions 2 and 3).

An analysis of variance (ANOVA) with repeated-measures factors Measurement Occasion (Measurement 1 vs. Measurement 2 vs. Measurement 3) and Detection Task (Erroneous Counts vs. Pseudoerrors without Cardinal Value vs. Pseudoerrors with Cardinal Value) was performed on correct responses. The main effects of Measurement Occasion, $F[2,46] = 13.85$, $p < .01$, $\eta_p{}^2 = .38$, and Detection Task, $F[2,46] = 45.01$, $p < .01$, $\eta_p{}^2 = .66$, and their interaction, $F[4,92] = 3.65$, $p <$ 01, $\eta_p{}^2 = .14$, were significant (see Table 2 and Fig. 1).

To further analyze the interaction, separate one-way ANOVAs were performed for each measurement occasion. These revealed effects of Measurement 1, $F[2,46] = 18.34$, $p < .001$, $\eta_p{}^2 = .44$, Measurement 2, $F[2,46] = 29.01$, $p < .001$, $\eta_p 2 = .56$, and Measurement 3, $F[2,46] = 25.76$, $p < .001$, $\eta_p{}^2 = .53$. At Measurement 1, Bonferroni post hoc tests indicated significant differences in detection of Erroneous Counts and Pseudoerrors with and without Cardinal Value ($p < .001$ in both cases). At Measurements 2 and 3, multiple comparison analysis showed significant differences between Erroneous Counts and Pseudoerrors with and without Cardinal Value ($p < .001$ in both cases) and between the detection of Pseudoerrors without Cardinal Value and Pseudoerrors with Cardinal Value ($p < .05$).

Thus, while children detected Erroneous Counts better than Pseudoerrors at the three measurement occasions, children only showed significantly better detection of Pseudoerrors with Cardinal Value than those without Cardinal Value at occasions

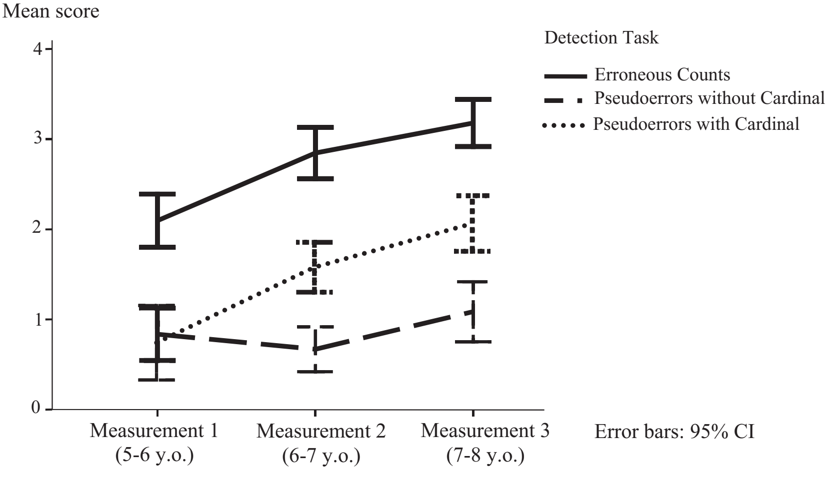

Fig. 1. Measurement Occasion and Detection Task interaction.

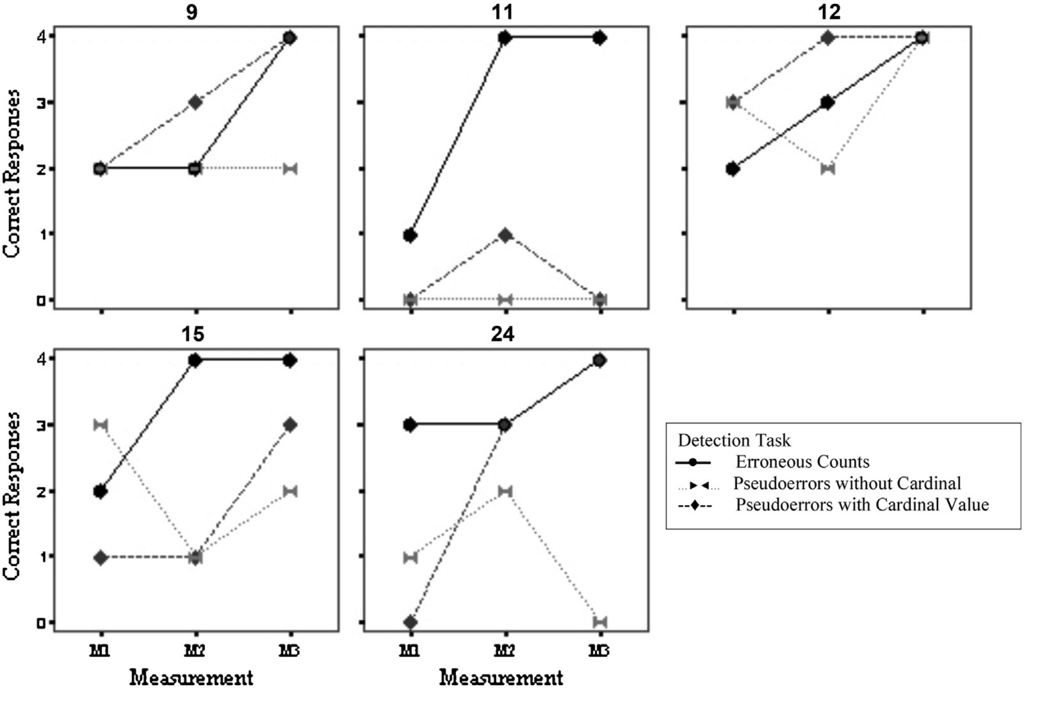

Fig. 2. Performance of a selection of five different participants on the Detection Tasks across the three Measurement Occasions.

2 and 3. Explicitly mentioning the cardinal values after unusual counts did not facilitate correct detection in 5–6-year-olds, possibly because many of them were not aware of the correctness of the result offered after counting. Indeed, at Measurement 1, participants tended to view the cardinal value stated with pseudoerrors as incorrect in 62.5% of the trials, which decreased to 26% at Measurement 2 and 13.5% at Measurement 3. At Measurement 1, children believed that obtaining a correct cardinal value depended both on the correct application of the one-to-one correspondence and stable-order principles as well as on conventional rules (e.g., spatial adjacency). The response offered by Laura (age 65 months) for *Pseudoerror 6* with Cardinal Value (counting by alternating elements) was representative: "Counting like that, she can't know how many there are (. . .) To know how many balls there are, she had to count them in order, like this [indicating with her finger from left to right]." From Measurement 2, although children still rejected transgressions of the conventional rules, they realized that the cardinal value was tied to logical rules, such as counting all elements once. This change is well illustrated in the response of the same girl on *Pseudoerror 6* at Measurement 3 (age 89 months): "There are 12 balls, but she hasn't counted right because you can't count the soccer balls and then the basketballs. You have to count them one by one, one after another."

Children's ability to correctly detect pseudoerrors seemed to proceed through distinct changes. Those with cardinal value followed a linear development, but there were no significant differences for pseudoerrors without cardinal value. The finding that children's ability to correctly identify and understand conventional rules develops gradually with age agrees with the findings of authors such as Briars and Siegler (1984), Rodríguez et al. (2013), and Saxe et al. (1989).

Regarding individual patterns, longitudinal data revealed (a) three different developmental trajectories – constant or flat, linear, and quadratic – regardless of the task (see Fig. 2) and (b) initial performance level did not affect the developmental course followed.

The first trajectory encompasses participants whose performance showed no change over subsequent measurement occasions. Examples include the performance of participants 9, and 11 in Pseudoerrors without Cardinal Value (see Fig. 2). The linear trajectory includes children whose performance systematically increased over the three measurement occasions. However, although the children became more precise in judging counts as they grew older, they did not progress in the same manner or at similar paces. For example, the improvement in detecting Pseudoerrors with Cardinal Value was more pronounced in participant 24 than in participant 9. Finally, in the quadratic trend, depending on the interval during which children's performance decreased ($T1 \geq T2 \leq T3$ or $T1 \leq T2 \geq T3$), two developmental patterns emerged, either U-shaped (e.g., participant 15 in the Pseudoerrors without Cardinal Value) or reversed U-shaped (e.g., participant 11 in the Pseudoerrors with Cardinal Value). The quadratic trend also encompassed participants whose performances remained stable between consecutive measurements ($T1 \leq T2 \leq T3$ or $T1 \geq T2 \geq T3$; for example, see participants 15 and 24 in the Erroneous Counts). Taking this into account, analyses of the estimation models (linear, quadratic, or constant) were undertaken to assess the best fit for each participant on each task. Only asymptotic ceiling curves were excluded to avoid distortion of the data. Table 3 lists the frequency of the estimated curves

Table 3
Curvilinear regression analysis estimates for participants on each of the counting detection tasks.

Curvilinear Estimation Model	Erroneous Counts	Pseudoerrors without Cardinal Value	Pseudoerrors with Cardinal Value
Linear	3	0	3
Quadratic	13	16	13
Constant (flat)	3	7	5

for each of the tasks and shows that the quadratic model best represented the performance of the majority of participants on the three detection tasks.

3. Discussion

Results of this study show that children require several years to understand the optional nature of the conventional rules of counting. Additionally, as expected, the detection of erroneous counts was easier than the detection of pseudoerrors. Although the errors and pseudoerrors we employed differed from those used in the majority of previous studies, the data agree with the findings of Briars and Siegler (1984) with 3–5-year-olds and of Geary et al. (1992, 2000, 2004), Escudero (2012), Kamawar et al. (2010), LeFevre et al. (2006), and Rodríguez et al. (2013) with older children.

Another noteworthy finding is that the relevance that children ascribed to the conventional rules varied based on the type of task. The children became more tolerant of the conventional rules when logical rules were also violated. For instance, Juan (age 85 months) for *Error 2* noted: “It is wrong because she—Eva—has started from 1 again when she counted the blunt pencils and there aren’t 9 pencils in total.” In erroneous trials, children judged the counting to be incorrect due to the violation of logical rules not because of the transgression of conventional rules. Moreover, not all pseudoerrors presented the same difficulty for participants. At Measurement 2, there were significant differences in the children’s rates of success depending on whether pseudoerrors were presented with or without the cardinal value. The presence of the cardinal value significantly increased the probability that children would accept the pseudoerrors as valid counts.

Two possible reasons that explain the positive effect of the cardinal value in pseudoerrors are (a) that the presence of the cardinal value contributed to minimizing the importance ascribed to the conventional rules because the children focused on the functional aspect of counting, thereby making the unusual counting procedure irrelevant (Rodríguez et al., 2013), or (b) that children performed their own subvocal counting. We believe that the second possibility poses at least the following difficulties: If children had counted subvocally, they would have judged all of the pseudoerrors (with and without cardinal values) as correct due to matching with the cardinal value even when the character did not explicitly say, “There are ___ [number of objects].” Similarly, their justifications would be based on this match, without any reference to conventional rules. Nevertheless, results of the present study revealed differences between pseudoerrors with and without cardinal values and demonstrated that children’s incorrect justifications were based on conventional rules. Moreover, evidence favoring the first explanation includes the fact that presence of the cardinal value did not begin to exert a clear effect until children had consolidated an understanding of cardinality (after 6–7 years of age).

Mastery of the ability to count is not as simple or rapid as one might assume. How then does this knowledge develop? The present results may help to answer this question. On the one hand, data from the group analysis indicate that changes in the understanding of logical and conventional rules of counting are linear and gradual in the early primary school grades. These are the years when children begin to learn the algorithms of arithmetic that significantly increase opportunities to practice counting and provide a sound framework not only to practice logical rules but also to highlight and promote the optional nature of conventional rules (e.g., the children build strategies, such as counting backwards and starting to count from the larger addend to solve addition and subtraction problems). Indeed, studies by Geary et al. (1992, 2000, 2004) and Geary (2011) reported data on the relation between successful detection of pseudoerrors and use of certain arithmetical strategies, such as decomposition and the min-procedure (counting from the larger addend), to solve addition problems.

On the other hand, results from individual analysis suggest the presence of important individual differences in the comprehension of logical and conventional rules. Participants differed in their levels of initial and final knowledge as well as in their rates of change, allowing us to propose different developmental trajectories, consistent with results from other domains of mathematical thinking (e.g., understanding fractions or arithmetic knowledge; Dowker, 2008; Hallett et al., 2012; Jordan et al., 2009). The quadratic model represented the best fit for children's performance; this result might be interpreted in light of the *change resistance approach* (Luchins & Luchins, 1950; McNeil, 2007; McNeil & Alibali, 2005). These authors maintain that previous learning may constrain or slow new learning; that is, given that, during the learning process, children not only incorporate new concepts and/or procedures as they advance in formal schooling but must also relate these concepts to those already learned, there could be interferences, especially in novel tasks, such as the ones employed in this study. This aspect should be explored further, since our conclusions are limited by the modest size of our sample (24 participants). The constraints of a three-year longitudinal design make this sample valid and sufficient to produce clear and significant results. However, a bigger sample would make it possible to extend the findings by revealing, for example, other developmental patterns or more marked individual differences.

Finally, from an educational perspective, procedural knowledge has traditionally received much attention in the teaching of mathematics; nevertheless this knowledge plays an important role in children's failures when faced with non-habitual tasks. In effect, the process of applying a procedure (e.g., counting) is different from the process of reflecting on that procedure (e.g., judging whether a character has employed it correctly or incorrectly). Emphasis on this second aspect tends to be ignored in the teaching of mathematics and replaced by a stereotyped array of tasks. This might explain the lack of success of our participants in detecting pseudoerrors. A priori, a counting task should be easy, but it ceases to be so (even for second graders) when the procedure does not match the conventional rules.

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